M118
FALL 2000

QUIZ 6

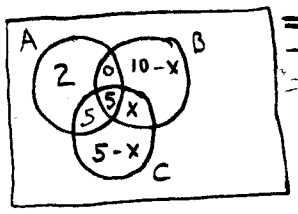
NAME _____

Due: Monday, October 9.

1) Let A and B be sets. If $n(A) = 4$ and $n(B) = 3$ then $n(A \times B) = ?$

Answer: 12

2) Let A, B and C be subsets of a universal set U. If $n(A \cup B \cup C) = 25$, $n(A \cap B) = 5$, $n(B) = 15$, and $n(A \cap C) = 10$, $n(A \cap B \cap C) = 5$, $n(A) = 12$, $n(C) = 15$ then $n(B \cap C) = ?$.



$\Rightarrow 12 + 10 - X + X + 5 - X = 25 = n(A \cup B \cup C) \Rightarrow -X = -2 \quad X = 2$

$\Rightarrow n(B \cap C) = 7$

OR USE

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$25 = 12 + 15 + 15 - 10 - 5 - ? + 5$

$\Rightarrow ? = 7$

Answer: 7

3) Let A and B be subsets of a universal set U. If $n(A \cup B) = 25$, $n(U) = 28$, $n(A) = 15$, and $n(A \cap B) = 10$ then $n(B') = ?$.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$25 = 15 + n(B) - 10$

$\Rightarrow n(B) = 20 \Rightarrow n(B') = 28 - 20 = 8$

Answer: 8.

4) Which of the following statements are true for any subsets A and B of a universal set U?

(i) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$ ✓ $\Leftrightarrow$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $A \subset A \cap B$ ✗

(iii) $A \cup \emptyset = A$ ✓

(iv) $(A \cup B)' = A' \cap B'$ ✓ de Morgan's law

Answer: i), iii) + iv)

5) Which of the following statements are true for any sets A, B, and C?

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ ✓

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ ✓

(iii) $n(A \times B \times C) = n(A) \cdot n(B) \cdot n(C)$ ✓

(iv) $(A \cap B) \cup C = A \cap (B \cup C)$ ✗

Answer: i) ii) iii)

SEE OTHER SIDE

6) An Urn contains 15 blue , 4 red, 17 black, and 9 white balls. One ball is drawn at random. What is the probability that the ball is white or red?

$\frac{13}{45}$ ← # WorR / # balls

Answer: $\frac{13}{45}$

7) A danish Professor must select 4 different areas of mathematics to discuss in a course. He has to select at most two areas from algebra. In how many different ways may he select the 4 areas from a group of areas that consists of 6 areas in analysis and 4 areas in algebra?

ways to select 2 algebra + 2 analysis = $\binom{6}{2}\binom{4}{2} = 15 \cdot 6 = 90$

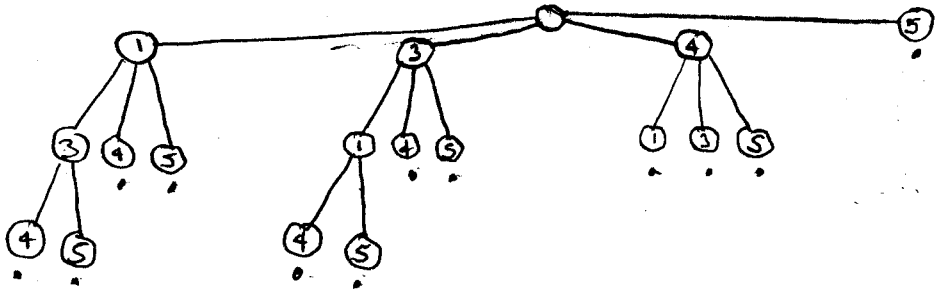
ways to select 1 algebra + 3 analysis = $\binom{6}{3}\binom{4}{1} = 20 \cdot 4 = 80$

ways to select 0 algebra + 4 analysis = $\binom{6}{4} = 15$

$90 + 80 + 15 = 185$

Answer: 185

8) An urn contains 4 slips of papers numbered 1, 3, 4, and 5. An experiment consists of successively drawing slips of paper from the urn without replacement and recording the numbers until the sum of the numbers drawn is at least 5. As soon as the sum of the numbers drawn is at least 5, the experiment ends. How many different outcomes are possible in this experiment?



12 dots

Answer: 12

9) Three different cars (Ford, Oldsmobile, and Pontiac) have to be painted. Each car will be painted with one color. (That is, a car can not be painted with two colors.) There are 4 colors of paint which may be used (yellow, pink, blue, and black). A car decoration scheme is a specification of a color for each the three cars. How many different decoration schemes use at least two different colors of paint?

$4 \cdot 4 \cdot 4 - 4 = 64 - 4 = 60$

$4 \cdot 4 \cdot 4$: # schemes with no restrictions

4: # schemes that don't use 2 or more colors i.e. all same color

Answer: 60