

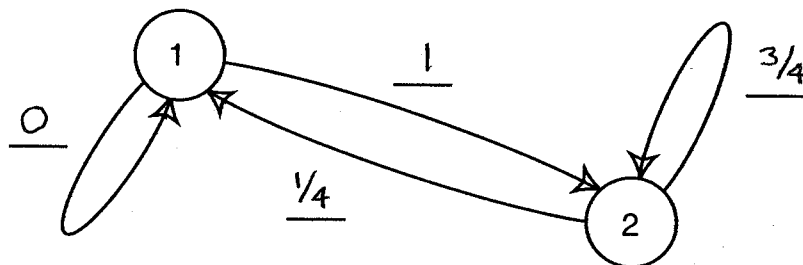
QUIZ 9

NAME _____

- 1) Given below is, P , the transition matrix for a Markov process with two states (states 1 and 2):

$$\begin{matrix} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \end{matrix}$$

- a) (15 PTS.) Fill in the blanks below, so that the diagram given below is the transition diagram that corresponds to the transition matrix given above.



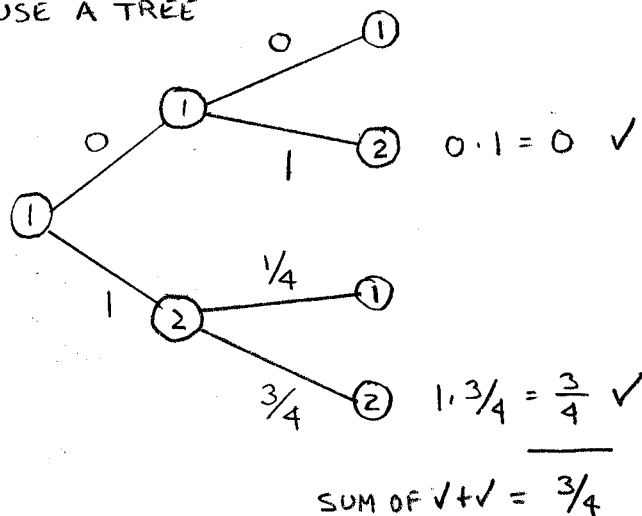
- b) (15 PTS.) Find the probability that the system will be in state two after 2 transitions, if it starts out in state 1.

$$P_{1,2}(2) = (P^2)_{1,2} = \left[\begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \right]_{1,2} = \left[\begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{16} & \frac{13}{16} \end{pmatrix} \right]_{1,2}$$

1,2 ENTRY = $\frac{3}{4}$

ANSWER: $\frac{3}{4}$

OR USE A TREE



Answer: $Pr[\text{State2}] = \frac{3}{4}$

1) cont'd $P = \begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

c) (15 PTS.) Find a stable vector $w = (w_1, w_2)$ for the transition matrix P .

$$(x, 1-x) \begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} = (x, 1-x)$$

$$\Rightarrow x \cdot 0 + (1-x) \cdot \frac{1}{4} = x$$

$$\Rightarrow 0 + \frac{1}{4} - \frac{1}{4}x = x$$

$$\Rightarrow \frac{1}{4} = \frac{5}{4}x$$

$$\Rightarrow x = \frac{1}{5} \Rightarrow 1-x = \frac{4}{5}$$

CHECK: $(\frac{1}{5}, \frac{4}{5}) \begin{pmatrix} 0 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} = (\frac{1}{5}, \frac{4}{5})$

Answer: $w = (\frac{1}{5}, \frac{4}{5})$

d) (5 PTS.) Find a stable vector, $w = (w_1, w_2)$, for P^2 .

$$\text{If } wP = w, \text{ then } wP^2 = w(P P) = (wP)P = wP = w.$$

ANSWER: SAME AS PART c), $(\frac{1}{5}, \frac{4}{5})$

Answer: $w = (\frac{1}{5}, \frac{4}{5})$