M118
FALL 2000

QUIZ 15

NAME _____

1) a) (10 PTS.) A society produces two goods* - alfalfa and wheat. The technology matrix for this set-up is given below.

$$\begin{array}{c} \\ A \\ W \end{array}\begin{array}{c} \begin{array}{cc} A & W \end{array} \\ \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix} \end{array}$$

How many units of alfalfa are required to produce one unit of wheat?

Answer: .3

b) (25 PTS.) There is an external demand for 15 units of alfalfa and 18 units of wheat. To meet this requirement what should the production schedule be (i.e. how many unit of alfalfa and wheat should be produced)?

$A = \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix}$ $D = \begin{pmatrix} 15 \\ 18 \end{pmatrix}$ $I-A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix} = \begin{pmatrix} .5 & -.3 \\ 0 & .9 \end{pmatrix}$

$(I-A)X = D \Rightarrow$ ★ $\left(\begin{array}{cc|c} .5 & -.3 & 15 \\ 0 & .9 & 18 \end{array}\right) \xrightarrow{\frac{1}{3}R_2} \left(\begin{array}{cc|c} .5 & -.3 & 15 \\ 0 & .3 & 6 \end{array}\right)$

($I-A$ | D)

$\xrightarrow{R_2+R_1} \left(\begin{array}{cc|c} .5 & 0 & 21 \\ 0 & .3 & 6 \end{array}\right) \xrightarrow[\frac{10}{3}\cdot R_2]{2\cdot R_1} \left(\begin{array}{cc|c} 1 & 0 & 42 \\ 0 & 1 & 20 \end{array}\right)$

★ Form the associated augmented matrix

Answer: Alfalfa 42

Wheat 20

*This is actually a farm - wheat and alfalfa are needed to produce seeds for further production. Alfalfa is grown as part of a crop rotation system to replenish the soil.

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1) a) (10 PTS.) A society produces two goods* - alfalfa and wheat. The technology matrix for this set-up is given below.

$$\begin{array}{c} \\ A \\ W \end{array}\begin{array}{c} \begin{array}{cc} A & W \end{array} \\ \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix} \end{array}$$

How many units of alfalfa are required to produce one unit of wheat?

Answer: .3

b) (25 PTS.) There is an external demand for 2 units of alfalfa and 18 units of wheat. To meet this requirement what should the production schedule be (i.e. how many unit of alfalfa and wheat should be produced)?

$A = \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix}$ $D = \begin{pmatrix} 2 \\ 18 \end{pmatrix}$ $I - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix} = \begin{pmatrix} .5 & -.3 \\ 0 & .9 \end{pmatrix}$

$(I-A)X = D \Rightarrow$ ✱ $\left(\begin{array}{cc|c} .5 & -.3 & 2 \\ 0 & .9 & 18 \end{array}\right) \xrightarrow{1/3\,R_2} \left(\begin{array}{cc|c} .5 & -.3 & 2 \\ 0 & .3 & 6 \end{array}\right)$

($I-A$ | D)

$\xrightarrow{R_2 + R_1} \left(\begin{array}{cc|c} .5 & 0 & 8 \\ 0 & .3 & 6 \end{array}\right) \xrightarrow[\frac{10}{3}\cdot R_2]{2\cdot R_1} \left(\begin{array}{cc|c} 1 & 0 & 16 \\ 0 & 1 & 20 \end{array}\right)$ (X)

✱ Form the associated augmented matrix.

Answer: Alfalfa 16

Wheat 20

*This is actually a farm - wheat and alfalfa are needed to produce seeds for further production. Alfalfa is grown as part of a crop rotation system to replenish the soil.

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QUIZ 15

NAME _____

1) a) (10 PTS.) A society produces two goods* - alfalfa and wheat. The technology matrix for this set-up is given below.

$$\begin{array}{c} \\ A \\ W \end{array}\begin{array}{c} \begin{array}{cc} A & W \end{array} \\ \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix} \end{array}$$

How many units of wheat are required to produce one unit of alfalfa?

Answer: 0

b) (25 PTS.) There is an external demand for 12 units of alfalfa and 18 units of wheat. To meet this requirement what should the production schedule be (i.e. how many unit of alfalfa and wheat should be produced)?

$A = \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix}$ $D = \begin{pmatrix} 12 \\ 18 \end{pmatrix}$ $I - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} .5 & .3 \\ 0 & .1 \end{pmatrix} = \begin{pmatrix} .5 & -.3 \\ 0 & .9 \end{pmatrix}$

$(I-A)X = D \Rightarrow$ ✡ $\left(\begin{array}{cc|c} .5 & -.3 & 12 \\ 0 & .9 & 18 \end{array}\right)$ ($I-A$ | D) $\xrightarrow{\frac{1}{3}R_2}$ $\left(\begin{array}{cc|c} .5 & -.3 & 12 \\ 0 & .3 & 6 \end{array}\right)$

$\xrightarrow{R_2 + R_1}$ $\left(\begin{array}{cc|c} .5 & 0 & 18 \\ 0 & .3 & 6 \end{array}\right)$ $\xrightarrow[\frac{10}{3}\cdot R_2]{2R_1}$ $\left(\begin{array}{cc|c} 1 & 0 & 36 \\ 0 & 1 & 20 \end{array}\right)$

✡ Form the associated augmented matrix.

Answer: Alfalfa 36

Wheat 20

*This is actually a farm - wheat and alfalfa are needed to produce seeds for further production. Alfalfa is grown as part of a crop rotation system to replenish the soil.

SEE OTHER SIDE

2) (15 PTS.) Use the same set-up as in problem 1). The production schedule calls for 600 units of alfalfa and 100 units of wheat. What is the external demand (i.e. what is the excess production)?

$(I - A)X = D$

$$\underbrace{\begin{pmatrix} .5 & -.3 \\ 0 & .9 \end{pmatrix}}_{I-A}\underbrace{\begin{pmatrix} 600 \\ 100 \end{pmatrix}}_{X} = \begin{pmatrix} 270 \\ 90 \end{pmatrix}$$

This is just matrix multiplication

Answer: Alfalfa 270

Wheat 90

2) (15 PTS.) Use the same set-up as in problem 1). The production schedule calls for 200 units of alfalfa and 100 units of wheat. What is the external demand (i.e. what is the excess production)?

$$(I-A)\,X = D$$

$$\underbrace{\begin{pmatrix} .5 & -.3 \\ 0 & .9 \end{pmatrix}}_{(I-A)} \underbrace{\begin{pmatrix} 200 \\ 100 \end{pmatrix}}_{X} = \begin{pmatrix} 70 \\ 90 \end{pmatrix} = D$$

This is just matrix multiplication

Answer: Alfalfa 70

Wheat 90