M118
FALL 2000

QUIZ 13

NAME _____

1) a) (15 PTS.) Row reduce the following augmented matrix:

$$\begin{pmatrix} 0 & 0 & 2 & 0 & -1 & 2 \\ 1 & 1 & 8 & 1 & 1 & C \end{pmatrix}$$

What is the last column of this row reduced matrix?

$$R_1 \longleftrightarrow R_2 \quad \begin{pmatrix} 1 & 1 & 8 & 1 & 1 & C \\ 0 & 0 & 2 & 0 & -1 & 2 \end{pmatrix}$$

$$\xrightarrow{-4R_2+R_1} \begin{pmatrix} 1 & 1 & 0 & 1 & 5 & -8+C \\ 0 & 0 & 2 & 0 & -1 & 2 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} 1 & 1 & 0 & 1 & 5 & -8+C \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 1 \end{pmatrix} \quad \star$$

3 version: $C = 4, 5, 6$

Answers: $\begin{pmatrix} -4 \\ C \end{pmatrix}, \begin{pmatrix} -3 \\ C \end{pmatrix}, \begin{pmatrix} -2 \\ C \end{pmatrix}$

Answer: The <u>LAST</u> column is $\begin{pmatrix} -8+C \\ 1 \end{pmatrix}$

b) (15 PTS.) Consider the following system of equations:

$$\begin{aligned} 0x_1 + 0x_2 + 2x_3 + 0x_4 - 1x_5 &= 2 \\ 1x_1 + 1x_2 + 8x_3 + 1x_4 + 1x_5 &= 6 \end{aligned}$$

Solve these equations for x_1 in terms of the free variables.

The augmented matrix for this system is the matrix given in part a) So row reducing to $\star$, and rewriting the top line of $\star$ as an equation yields

$$x_1 + x_2 + x_4 + 5x_5 = -8+C$$

$$\Rightarrow \quad x_1 = -x_2 - x_4 - 5x_5 - 8 + C$$

Fill in the blanks with numbers (some of which may be zero):

$$x_1 = \underline{(-1)}\, x_{\underline{2}} + \underline{(-1)}\, x_{\underline{4}} + \underline{(-5)}\, x_{\underline{5}} + \underline{\overbrace{-8+C}^{-4,\,-3,\,6}}$$

2) (20 PTS.) Row reduce the following augmented matrix:

$$\begin{pmatrix} 0 & 0 & 1 & 6 \\ 0 & 4 & 1 & 4 \\ 1 & 2 & 0 & c \end{pmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 0 & c \\ 0 & 4 & 1 & 4 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

3 versions:

$c = 5, 6, 7$

$$\xrightarrow{-R_3 + R_2} \begin{pmatrix} 1 & 2 & 0 & c \\ 0 & 4 & 0 & -2 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} 1 & 2 & 0 & c \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

$$\xrightarrow{-R_2 + R_1} \begin{pmatrix} 1 & 0 & 0 & c+1 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 0 & c+1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

$c+1 = 6, 7, \text{ or } 8.$

Answer (fill in the blanks):

$$\begin{pmatrix} 1 & 0 & 0 & c+1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 6 \end{pmatrix}$$