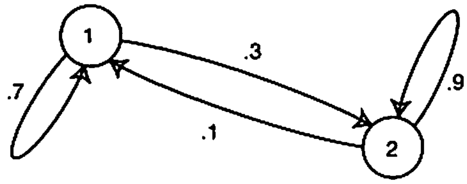
M118
FALL 09

QUIZ 10

NAME ____________________

1) Shown below is the transition diagram for a Markov process with two states (states 1 and 2):



a) (10 PTS.) Let the transition matrix for this process be P. Fill in the blanks below for the entries of P.

$$P = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & .7 & .3 \\ 2 & .1 & .9 \end{array}$$

$$\begin{pmatrix} .7 & .3 \\ .1 & .9 \end{pmatrix}\begin{pmatrix} .7 & .3 \\ .1 & .9 \end{pmatrix} = \begin{pmatrix} .52 & .48 \\ .16 & .84 \end{pmatrix}$$

b) (10 PTS.) Fill in the blanks:

c) ↓

$$P^2 = \begin{pmatrix} .52 & .48 \\ .16 & .84 \end{pmatrix}$$

c) ⟶

c) (10 PTS.) Find the probability that the system will be in **state 1** after TWO transitions, if it starts out in **state 2**.

See the (2,1) entry of P^2

Answer: $Pr[State\ 1] =$ .16

d) (10 PTS.) Find the probability that the system will be in **state 1** after TWO transitions, if it starts out with an initial state vector of $(1/2, 1/2)$. That is, initially there is a 1/2 probability that the system is in state 1, and a 1/2 probability that it is in state 2.

$$(1/2, 1/2)\begin{pmatrix} .52 & .48 \\ .16 & .84 \end{pmatrix} = \left(\tfrac{1}{2}\cdot .52 + \tfrac{1}{2}\cdot .16,\ \sim\right) = (.34, \sim)$$

Answer: $Pr[State\ 1] =$ .34

e) (10 PTS.) Find the probability that if the system starts out in state 1, then in the next two transitions it will be in state 2 then state 1. In other words, $Pr[1 \to 2 \to 1] = ?$

$$1 \xrightarrow{.3} 2 \xrightarrow{.1} 1 \qquad .3 \cdot .1 = .03$$

Answer: $Pr[State\ 1] =$ .03

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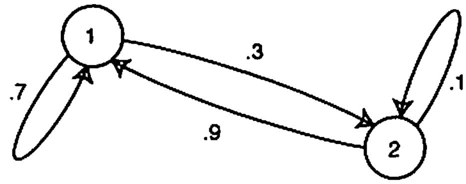
1) Shown below is the transition diagram for a Markov process with two states (states 1 and 2):



a) (10 PTS.) Let the transition matrix for this process be P. Fill in the blanks below for the entries of P.

$$P = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} \underline{.7} & \underline{.3} \\ \underline{.9} & \underline{.1} \end{pmatrix} \end{array}$$

$$\begin{pmatrix} .7 & .3 \\ .9 & .1 \end{pmatrix}\begin{pmatrix} .7 & .3 \\ .9 & .1 \end{pmatrix} = \begin{pmatrix} .76 & .24 \\ .72 & .28 \end{pmatrix}$$

b) (10 PTS.) Fill in the blanks:

c) $\downarrow$

c) $\longrightarrow$ $P^2 = \begin{pmatrix} .76 & .24 \\ .72 & .28 \end{pmatrix}$

c) (10 PTS.) Find the probability that the system will be in **state 1** after TWO transitions, if it starts out in **state 2**.

See (2,1) entry of P^2

Answer: $Pr[State\ 1] =$ $\underline{.72}$

d) (10 PTS.) Find the probability that the system will be in **state 1** after TWO transitions, if it starts out with an initial state vector of $(1/2, 1/2)$. That is, initially there is a $1/2$ probability that the system is in state 1, and a $1/2$ probability that it is in state 2.

$$(\tfrac{1}{2}, \tfrac{1}{2})\begin{pmatrix} .76 & .24 \\ .72 & .28 \end{pmatrix} = (\tfrac{1}{2}\cdot .76 + \tfrac{1}{2}\cdot .72, \sim) = (.74, \sim)$$

Answer: $Pr[State\ 1] =$ $\underline{.74}$

e) (10 PTS.) Find the probability that if the system starts out in state 1, then in the next two transitions it will be in state 2 then state 1. In other words, $Pr[1 \to 2 \to 1] = ?$

$$1 \xrightarrow{.3} 2 \xrightarrow{.9} 1 \qquad .3 \cdot .9 = .27$$

Answer: $Pr[State\ 1] =$ $\underline{.27}$