

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting 4 aces?

ERASE



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$$\Pr[4 \text{ aces}] = \frac{X}{C(52,5)}$$

of ways to pick 5 cards with 4 of them being aces.

of ways to pick any 5 cards

Lecture 7

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QUESTION: How many 5 card hands have 4 aces (i.e. what is X)?

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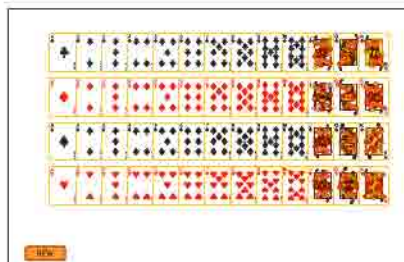
ERASE



4



QUESTION: How many 5 card hands have 4 aces (i.e. what is X)?



To see this, lay the deck out on a table, and start making up all the 5 card hands with 4 aces. First, you will want to select the 4 aces.

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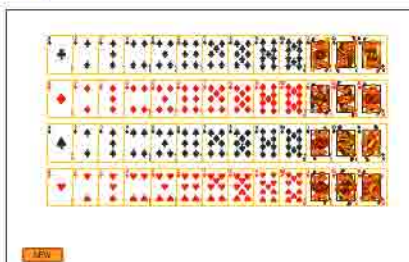
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$$X = 48$$

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$$\Pr[4 \text{ aces}] = \frac{X}{C(52,5)} = \frac{48}{C(52,5)}$$

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$$\Pr[4 \text{ aces}] = \frac{X}{C(52,5)} = \frac{48}{C(52,5)}$$

$$= \frac{48}{\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{48}{2,598,960} = \frac{1}{54145}$$

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a full house?

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[Full house: 3 cards of one value and 2 cards of another.]

$$\Pr[\text{Full house}] = \frac{X}{C(52,5)}$$

of hands with 3 cards of one value and 2 of another.

of ways to pick any 5 cards

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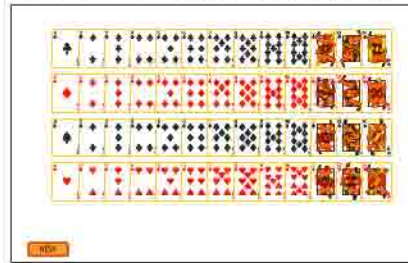
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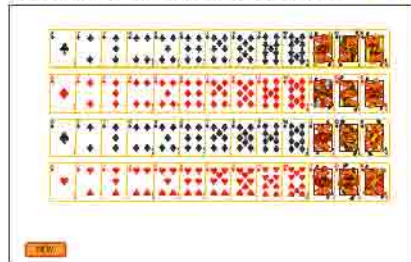
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QUESTION: How many 5 card hands have 3 cards of one value and 2 of another (i.e. what is X)?



1) Choose two values: $C(13,2)$ ways.

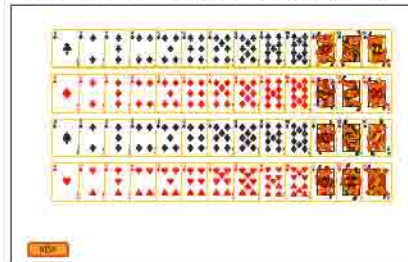
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QUESTION: How many 5 card hands have 3 cards of one value and 2 of another (i.e. what is X)?



1) Choose two values: $C(13,2)$ ways.
2) Pick one value for 3 of a kind: 2 ways.

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2) Pick one value for 3 of a kind: 2 ways.
3) Pick 3 of 4 cards of value just chosen: $C(4,3)$ ways.

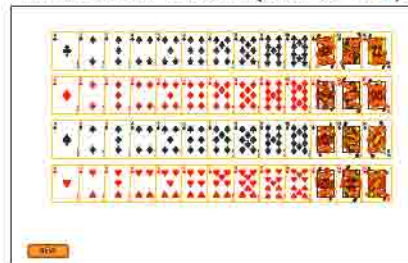
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4) Pick two cards other value: $C(4,2)$ ways.

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$X = C(13,2) \cdot 2 \cdot C(4,3) \cdot C(4,2)$
 $= 3,744$

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[Full house: 3 cards of one value and 2 cards of another value.]

$$\Pr[\text{Full house}] = \frac{X}{C(52,5)}$$

of hands with 3 cards of one value and 2 of another.

of ways to pick any 5 cards

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 $= 3,744$

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EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a full house?
 [Full house: 3 cards of one value and 2 cards of another value.]



$$\Pr[\text{Full house}] = \frac{X}{C(52,5)} = \frac{3,744}{2,598,960}$$

$$X = C(13,2) \cdot C(4,3) \cdot C(4,2) = \frac{6}{4165} = 3,744$$

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a flush?
 [Flush: All 5 cards the same suit.]



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EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a flush?
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SOLUTION:

$$\frac{X}{C(52,5)}$$

of 5 card hands with all cards the same suit.

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a flush?
 [Flush: All 5 cards the same suit.]



SOLUTION:

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of 5 card hands with all cards the same suit.

To find X, try making up all flush hands:

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To find X, try making up all flush hands:

- 1) Choose suit: 4 ways to do this

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SOLUTION:

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of 5 card hands with all cards the same suit.

To find X, try making up all flush hands:

- 1) Choose suit: 4 ways to do this
- 2) Choose 5 cards of that suit: $C(13,5)$ ways to do this

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 [Flush: All 5 cards the same suit.]



SOLUTION:

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of 5 card hands with all cards the same suit.

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- 1) Choose suit: 4 ways to do this
- 2) Choose 5 cards of that suit: $C(13,5)$ ways to do this
- 3) There are $X = 4 \cdot C(13,5)$ possible hands that are flushes.

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a flush?
 [Flush: All 5 cards the same suit.]



SOLUTION:

$$\frac{4 \cdot C(13,5)}{C(52,5)}$$

To find X, try making up all flush hands:

- 1) Choose suit: 4 ways to do this
- 2) Choose 5 cards of that suit: $C(13,5)$ ways to do this
- 3) There are $X = 4 \cdot C(13,5)$ possible hands that are flushes.

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[Flush: All 5 cards the same suit.]

SOLUTION:

$$\frac{4 C(13,5)}{C(52,5)}$$

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EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a flush?

[Flush: All 5 cards the same suit.]

SOLUTION:

$$\frac{4 C(13,5)}{C(52,5)} = \frac{4 \left[\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \right]}{\left[\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \right]}$$
$$= \frac{4 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} = \frac{33}{1,666}$$

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Lecture 7 ■ ■

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a pair?

[Pair: Exactly 2 cards match in value.]

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[Pair: Exactly 2 cards match in value.]

SOLUTION:

$$\frac{X}{C(52,5)}$$

of 5 card hands with 2 cards same value.

To find X, try making up all "pair" hands:

- 1) Choose 4 values: $C(13,4)$ ways.
- 2) Choose one of the 4 values for a pair: $C(4,1)$ ways.
- 3) Choose two cards of "pair" value: $C(4,2)$ ways.
- 4) Choose one card of "singleton" value: $C(4,1)$ ways.
- 5) Choose one card of "singleton" value: $C(4,1)$ ways.
- 6) Choose one card of "singleton" value: $C(4,1)$ ways.

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[Pair: Exactly 2 cards match in value.]

SOLUTION:

$$\frac{X}{C(52,5)} = \frac{C(13,4) C(4,1) C(4,2) C(4,1) C(4,1) C(4,1)}{715 \cdot 4 \cdot 6 \cdot 4 \cdot 4 \cdot 4}$$

To find X, try making up all "pair" hands:

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- 6) Choose one card of "singleton" value: $C(4,1)$ ways.

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