PROBABILITIES AND EQUALLY LIKELY OUTCOMES:

QUICK EXAMPLE: A hat contains 7 slips of paper, 5 are red and 2 are white. One slip is picked at random. Each slip is just as likely as any other to be picked. What is the probability that a red slip of paper is chosen?

PROBABILITIES AND EQUALLY LIKELY OUTCOMES:

QUICK EXAMPLE: A hat contains 7 slips of paper, 5 are red and 2 are white. One slip is picked at random. Each slip is just as likely as any other to be picked. What is the probability that a red slip of paper is chosen?

Solution: 5/7

Let R be the event that a red slip is chosen.

$$\Pr[R] = \frac{n(R)}{n(S)} = \frac{5}{7}$$

PROBABILITIES AND EQUALLY LIKELY OUTCOMES:

QUICK EXAMPLE: A hat contains 7 slips of paper, 5 are red and 2 are white. One slip is picked at random. Each slip is just as likely as any other to be picked. What is the probability that a red slip of paper is chosen.

Solution: 5/7

Let R be the event that a red slip is chosen.

$$\Pr[R] = \frac{n(R)}{n(S)} = \frac{5}{7}$$

$S = \{R_1, R_2, R_3, R_4, R_5, W_1, W_2\}$

$R = \{R_1, R_2, R_3, R_4, R_5\}$

PROBABILITIES AND EQUALLY LIKELY OUTCOMES:

QUICK EXAMPLE: A hat contains 7 slips of paper, 5 are red and 2 are white. One slip is picked at random. Each slip is just as likely as any other to be picked. What is the probability that a red slip of paper is chosen.

Solution: 5/7

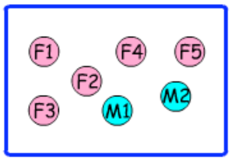
Let R be the event that a red slip is chosen.

$$\Pr[R] = \frac{n(R)}{n(S)}$$

\# of ways that things can happen the way you "want."

\# of ways anything can happen

EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that all three are female?



EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that all three are female?

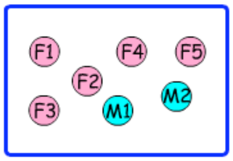
EXAMPLE: A room contains 5 females and 2 males. In how many ways can 3 of the people be selected (without regard to order)? 35

Different Question: How many ways can 3 be selected with only males or only females being selected? 10

Q2: How many ways with at least one male and one female? 25

Different Question: How many ways can 3 be selected with only females being selected? 10

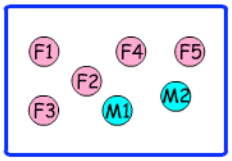
EXAMPLE: A room contains 5 females and 2 males. In how many ways can 2 of the females and 1 of the males be selected (without regard to order)? 20



EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that all three are female?

SOLUTION: We've already solved this. There are 35 ways to choose any 3 people, $C(7,3)$. There are 10 ways to pick all 3 female, $C(5,3)$.

So the sample space has 35 elements, and the event of picking all three females has 10 elements.

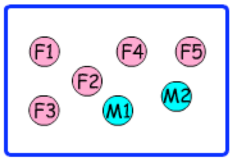


EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that all three are female?

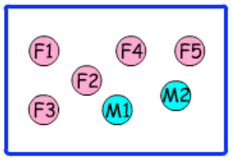
SOLUTION: We've already solved this. There are 35 ways to choose any 3 people, $C(7,3)$. There are 10 ways to pick all 3 female, $C(5,3)$.

So the sample space has 35 elements, and the event of picking all three females has 10 elements.

ANSWER: $\frac{10}{35} = \frac{2}{7}$



EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that at least one male and one female are chosen?



EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that at least one male and one female are chosen?

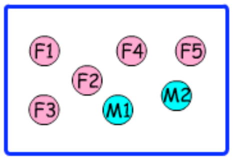
EXAMPLE: A room contains 5 females and 2 males. In how many ways can 3 of the people be selected (without regard to order)? 35

Different Question: How many ways can 3 be selected with only males or only females being selected? 10

Q2: How many ways with at least one male and one female? 25

Different Question: How many ways can 3 be selected with only females being selected? 10

EXAMPLE: A room contains 5 females and 2 males. In how many ways can 2 of the females and 1 of the males be selected (without regard to order)? 20

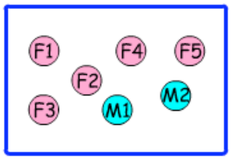


EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that at least one male and one female are chosen?

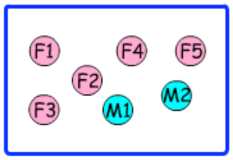
SOLUTION: We've already solved this. There are 35 ways to choose any 3 people, $C(7,3)$. There are 25 ways to pick 3 people such that at least one male and one female is chosen.

So the sample space has 35 elements, and the event of picking at least one male and one female has 25 elements.

ANSWER: $\frac{25}{35} = \frac{5}{7}$



EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that two females and one male are chosen?



EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that two females and one male are chosen?

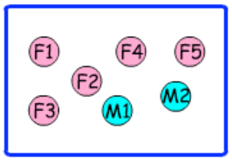
EXAMPLE: A room contains 5 females and 2 males. In how many ways can 3 of the people be selected (without regard to order)? 35

Different Question: How many ways can 3 be selected with only males or only females being selected? 10

Q2: How many ways with at least one male and one female? 25

Different Question: How many ways can 3 be selected with only females being selected? 10

EXAMPLE: A room contains 5 females and 2 males. In how many ways can 2 of the females and 1 of the males be selected (without regard to order)? 20

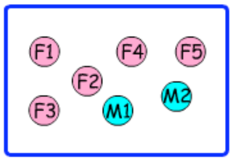


EXAMPLE: A room contains 5 females and 2 males. Three of these people are selected at random. What is the probability that two females and one male are chosen?

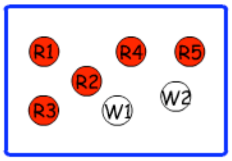
SOLUTION: We've already solved this. There are 35 ways to choose any 3 people, $C(7,3)$. There are $C(5,2) \cdot C(2,1) = 20$ ways to choose two females and one male.

So the sample space has 35 elements, and the event of picking 2 males and one female has 20 elements.

ANSWER: $\frac{20}{35} = \frac{4}{7}$



EXAMPLE: A hat contains 7 slips of paper, 5 are red and 2 are white. Three slips are picked at random. What is the probability that a two red slips and one white slip are chosen?

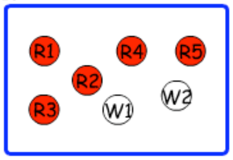


EXAMPLE: A hat contains 7 slips of paper, 5 are red and 2 are white. Three slips are picked at random. What is the probability that a two red slips and one white slip are chosen?

SOLUTION: We've already solved this, but with people not slips of paper. If you wish, you can number the slips of paper, just like the people.

So the sample space has 35 elements, and the event of picking 2 reds and one white slip has 20 elements.

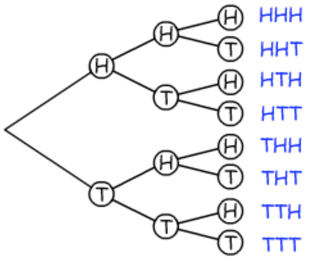
ANSWER: $\frac{20}{35} = \frac{4}{7}$



EXAMPLE: Flip a fair coin 3 times. What is the probability of getting exactly 2 heads?

EXAMPLE: Flip a fair coin 3 times. What is the probability of getting exactly 2 heads?

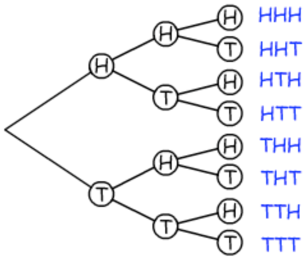
SOLUTION: The sample space for this procedure looks like:



S={HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

EXAMPLE: Flip a fair coin 3 times. What is the probability of getting exactly 2 heads?

SOLUTION: The sample space for this procedure looks like:

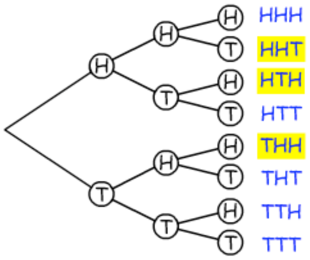


It has 8 elements. They are all equally likely.

S={HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

EXAMPLE: Flip a fair coin 3 times. What is the probability of getting exactly 2 heads?

SOLUTION: The sample space for this procedure looks like:



It has 8 elements. They are all equally likely. The event of getting exactly 2 heads has 3 elements.

ANSWER: $\frac{3}{8}$

S={HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

SOLUTION: Call the tourists A, B, C, D. For the first flight, assume the assignment is A1, B2, C3, D4. If it isn't then rename the tourists so that it is true. Now the question becomes:

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

SOLUTION: Call the tourists A, B, C, D. For the first flight, assume the assignment is A1, B2, C3, D4. If it isn't then rename the tourists so that it is true. Now the question becomes:

What is the probability that on the return flight, the seat assignment is A1, B2, C3, D4?

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

What is the probability that on the return flight, the seat assignment is A1, B2, C3, D4?

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION:

$$n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

Let E = the event lane 1 assigned in the first race.

Let F = the event lane 1 assigned in the second race.

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION:

$$n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

$n(E) = 8$ ⇐ There are 8 ways to assign lane one in the first race. Just assign lane one for the first race and any of lanes 1 thru 8 for the second.

Let E = the event lane 1 assigned in the first race.

Let F = the event lane 1 assigned in the second race.

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION:

$$n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

$n(E) = 8$

$n(F) = 8$ ⇐ Same reasonoing

Let E = the event lane 1 assigned in the first race.

Let F = the event lane 1 assigned in the second race.

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION:

$$n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

$n(E) = 8$

$n(F) = 8$

$n(E \cap F) = 1$ ⇐ Lane 1 both races, only one way to do this.

Let E = the event lane 1 assigned in the first race.

Let F = the event lane 1 assigned in the second race.

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION:

$$n(E \cup F) = \overset{8}{n(E)} + \overset{8}{n(F)} - \overset{1}{n(E \cap F)}$$

$n(E) = 8$

$n(F) = 8$

$n(E \cap F) = 1$ ⇐ Lane 1 both races, only one way to do this.

Let E = the event lane 1 assigned in the first race.

Let F = the event lane 1 assigned in the second race.

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION:

$$n(E \cup F) = \overset{8}{n(E)} + \overset{8}{n(F)} - \overset{1}{n(E \cap F)}$$

$$n(E \cup F) = 15$$

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION:

$$n(E \cup F) = \overset{8}{n(E)} + \overset{8}{n(F)} - \overset{1}{n(E \cap F)}$$

$$n(E \cup F) = 15$$

ANSWER: $\frac{15}{64}$

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION: The sample space looks like:

Race 2

		1	2	3	4	5	6	7	8
	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,8)
Race 1	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)	(4,8)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)	(5,8)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)	(6,8)
	7	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)	(7,8)
	8	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)	(8,6)	(8,7)	(8,8)

64 elements

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

What is the probability that on the return flight, the seat assignment is A1, B2, C3, D4?

How many seat assignments are possible?

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

What is the probability that on the return flight, the seat assignment is A1, B2, C3, D4?

How many seat assignments are possible? 4 people are put on a list. First on the list is assigned seat 1, second set 2, etc. Order matters. Answer: $P(4,4) = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

What is the probability that on the return flight, the seat assignment is A1, B2, C3, D4?

How many seat assignments are possible? 4 people are put on a list. First on the list is assigned seat 1, second set 2, etc. Order matters. Answer: $P(4,4) = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.

The assignment A1, B2, C3, D4 is just 1 of these 24 possible assignments. They are all equally likely.

EXAMPLE: 4 tourists are randomly assigned seats 1 through 4 on a flight to their vacation spot. On the return trip, they are again randomly assigned seats 1 through 4. What is the probability that all 4 sit in the same seats for both flights?

What is the probability that on the return flight, the seat assignment is A1, B2, C3, D4?

How many seat assignments are possible? 4 people are put on a list. First on the list is assigned seat 1, second set 2, etc. Order matters. Answer: $P(4,4) = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.

The assignment A1, B2, C3, D4 is just 1 of these 24 possible assignments. They are all equally likely.

ANSWER: $\frac{1}{24}$

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION: There are 64 possible lane assignments for the two races - 8 choices for the first race, 8 for the second - $8 \times 8 = 64$. How many of these assign lane 1 to the runner for one or more races?

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION: There are 64 possible lane assignments for the two races - 8 choices for the first race, 8 for the second - $8 \times 8 = 64$. How many of these assign lane 1 to the runner for one or more races?

Let E = the event lane 1 assigned in the first race.

Let F = the event lane 1 assigned in the second race.

What is $n(E \cup F)$?

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION: There are 64 possible lane assignments for the two races - 8 choices for the first race, 8 for the second - $8 \times 8 = 64$. How many of these assign lane 1 to the runner for one or more races?

Let E = the event lane 1 assigned in the first race.

Let F = the event lane 1 assigned in the second race.

What is $n(E \cup F) = n(E) + n(F) - n(E \cap F)$?

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION: The sample space looks like:

Race 2

		1	2	3	4	5	6	7	8
	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,8)
Race 1	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)	(4,8)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)	(5,8)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)	(6,8)
	7	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)	(7,8)
	8	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)	(8,6)	(8,7)	(8,8)

64 elements

15 elements assign lane 1 to the runner in at least one race.

EXAMPLE: A runner is assigned, at random, one of 8 lanes for a race. Then for a second race, she is assigned, at random, one of the 8 lanes. What is the probability that she gets lane 1 in at least one of the two races?

SOLUTION: The sample space looks like:

Race 2

		1	2	3	4	5	6	7	8
	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,8)
Race 1	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)	(4,8)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)	(5,8)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)	(6,8)
	7	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)	(7,8)
	8	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)	(8,6)	(8,7)	(8,8)

ANSWER: $\frac{15}{64}$

64 elements

15 elements assign lane 1 to the runner in at least one race.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

COMMENT: Drawing one set of 3 jars is just as likely as any other set of 3 jars. This problem can be solved using EQUALLY LIKELY EVENTS.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 2 PLUM AND 1 PEACH:
   a) Select 2 plum: There are $C(7,2)$ ways to do this.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 2 PLUM AND 1 PEACH:
   a) Select 2 plum: There are $C(7,2)$ ways to do this.
   THEN b) Select 1 peach: There are $C(8,1)$ ways to do this.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 2 PLUM AND 1 PEACH:
   a) Select 2 plum: There are $C(7,2)$ ways to do this.
   b) Select 1 peach: There are $C(8,1)$ ways to do this.
   c) For each of the $C(7,2)$ ways to select the 2 plum jars, there are $C(8,1)$ ways to proceed. SO, there are $C(7,2)\cdot C(8,1)$ ways to select 2 plum and 1 peach.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 2 PLUM AND 1 PEACH: $C(7,2)\cdot C(8,1)$

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 2 jars of plum and one jar of peach?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 2 PLUM AND 1 PEACH: $C(7,2)\cdot C(8,1)$
3) ANSWER:

$$\frac{C(7,2)\cdot C(8,1)}{C(20,3)}$$

# of ways to pick 2 plum & 1 peach

# of ways to pick any 3 jars

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting exactly 2 jars of plum?

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting exactly 2 jars of plum?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 2 PLUM AND 1 NOT PLUM: $C(7,2)\cdot C(13,1)$

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting exactly 2 jars of plum?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 2 PLUM AND 1 NOT PLUM: $C(7,2)\cdot C(13,1)$
3) ANSWER:

$$\frac{C(7,2)\cdot C(13,1)}{C(20,3)}$$

# of ways to pick 2 plum & 1 not plum

# of ways to pick any 3 jars

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 1 jar of plum, 1 of peach, and 1 of strawberry?

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 1 jar of plum, 1 of peach, and 1 of strawberry?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 1 OF EACH: $C(8,1)\cdot C(7,1)\cdot C(5,1)$ ways to do this.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random. What is the probability of getting 1 jar of plum, 1 of peach, and 1 of strawberry?

SOLUTION:

1) SELECT ANY 3: There are $C(20,3)$ ways to do this.
2) SELECT 1 OF EACH: $C(8,1)\cdot C(7,1)\cdot C(5,1)$ ways to do this.
3) ANSWER:

$$\frac{C(8,1)\cdot C(7,1)\cdot C(5,1)}{C(20,3)}$$

# of ways to pick 1 of each

# of ways to pick any 3 jars

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

$$= \frac{7\cdot8\cdot6}{20\cdot19\cdot18} = \frac{7\cdot8}{20\cdot19\cdot3} = \frac{7\cdot2}{5\cdot19\cdot3}$$

$$= \frac{14}{285}$$

# of ways to pick plum, peach, plum

6) ANSWER: $\dfrac{P(7,1)\cdot P(8,1)\cdot P(6,1)}{P(20,3)}$

# of ways to pick any 3 jars in order

EXAMPLE: You have 4 quarters and three dimes. You select, at random, 3 of the coins, one after another, without replacement.

Pr[Q then D then Q] = ?

1) 7 coins, select 3 keeping track of order: P(7,3) ways to do this.
2) 4 ways to select Q, then 3 ways to select D, then 3 ways to select Q. 4·3·3 = 36 ways to do this.

ANSWER:

$$\frac{36}{P(7,3)} = \frac{4\cdot3\cdot3}{7\cdot6\cdot5} = \frac{6}{35}$$

Pr[2Q & 1D any order] = ?

1) 7 coins, select 3 any order: C(7,3) ways to do this.
2) Select 2 Q: C(4,2) ways to do this
3) Select 1 D: C(3,1) ways.

ANSWER:

$$\frac{C(4,2)\cdot C(3,1)}{C(7,3)} = \frac{6\cdot3}{\left[\frac{7\cdot6\cdot5}{3!}\right]} = \frac{18}{35}$$

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man?

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man?

1) How many committees have exactly one women? Select one of the 3 women and 2 of the 4 men: C(3,1)·C(4,2) ways.

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man?

1) How many committees have exactly one women? Select one of the 3 women and 2 of the 4 men: C(3,1)·C(4,2) ways.
2) How many committees have exactly two women? Select two of the 3 women and 1 of the 4 men: C(3,2)·C(4,1) ways.

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man?

1) How many committees have exactly one women? Select one of the 3 women and 2 of the 4 men: C(3,1)·C(4,2) ways.
2) How many committees have exactly two women? Select two of the 3 women and 1 of the 4 men: C(3,2)·C(4,1) ways.

Answer: $C(3,1)\cdot C(4,2) + C(3,2)\cdot C(4,1) = 3\cdot 6 + 3\cdot 4 = 30$

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man?

30

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

SOLUTION:

1) SELECT ANY 3 IN ORDER: P(20,3) ways to do this.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

SOLUTION:

1) SELECT ANY 3 IN ORDER: P(20,3) ways to do this.
2) SELECT 1 PLUM: 7 ways to do this.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

SOLUTION:

1) SELECT ANY 3 IN ORDER: P(20,3) ways to do this.
2) SELECT 1 PLUM: 7 ways to do this.
3) NEXT SELECT 1 PEACH: 8 ways to do this.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

SOLUTION:

1) SELECT ANY 3 IN ORDER: P(20,3) ways to do this.
2) SELECT 1 PLUM: 7 ways to do this.
3) NEXT SELECT 1 PEACH: 8 ways to do this.
4) NEXT SELECT 1 PLUM: 6 ways to do this, since there are only 6 plum jars left.

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

SOLUTION:

1) SELECT ANY 3 IN ORDER: P(20,3) ways to do this.
2) SELECT 1 PLUM: 7 ways to do this.
3) NEXT SELECT 1 PEACH: 8 ways to do this.
4) NEXT SELECT 1 PLUM: 6 ways to do this, since there are only 6 plum jars left.
5) # OF WAYS TO SELECT PLUM, PEACH, PLUM: $7\cdot 8\cdot 6$

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

SOLUTION:

1) SELECT ANY 3 IN ORDER: P(20,3) ways to do this.
2) SELECT 1 PLUM: 7 ways to do this.
3) NEXT SELECT 1 PEACH: 8 ways to do this.
4) NEXT SELECT 1 PLUM: 6 ways to do this, since there are only 6 plum jars left.
5) # OF WAYS TO SELECT PLUM, PEACH, PLUM: $7\cdot 8\cdot 6$
6) **ANSWER:** $\dfrac{P(7,1)\cdot P(8,1)\cdot P(6,1)}{P(20,3)}$

EXAMPLE: A box contains 8 jars of peach preserves, 7 jars of plum preserves, and 5 jars of strawberry preserves. Three jars are drawn out at random, one after another. What is the probability of getting plum, then peach, then plum?

# of ways to pick plum, peach, plum

6) **ANSWER:** $\dfrac{P(7,1)\cdot P(8,1)\cdot P(6,1)}{P(20,3)}$

# of ways to pick any 3 jars in order

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

The question really asks: If one of these 30 committees is selected at random, what is the probability of getting Sam? "At random" means that each of the 30 committees is equally likely. How many of them have Sam? To see this, place Sam on the committee and select 2 more:

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

The question really asks: If one of these 30 committees is selected at random, what is the probability of getting Sam? "At random" means that each of the 30 committees is equally likely. How many of them have Sam? To see this, place Sam on the committee and select 2 more:

1) Two women: $C(3,2) = 3$ ways to do this.
2) One man, one woman $3 \cdot 3 = 9$ ways to do this.
3) $9 + 3 = 12$ possible committees with Sam.

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

The question really asks: If one of these 30 committees is selected at random, what is the probability of getting Sam?

3) $9 + 3 = 12$ possible committees with Sam.

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

The question really asks: If one of these 30 committees is selected at random, what is the probability of getting Sam?

ANSWER: $\frac{12}{30} = \frac{2}{5}$

3) $9 + 3 = 12$ possible committees with Sam.

EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

There are $C(2,1) = 2$ to select the woman (you can't select Sally again). There are $C(4,1) = 4$ ways to select 1 man. So there are $4 \cdot 2 = 8$ possible committees with Sally.

How many committees have at least one women and one man? 30

Sally?: Place Sally on the committee, then 2 men or 1 other woman and 1 man:

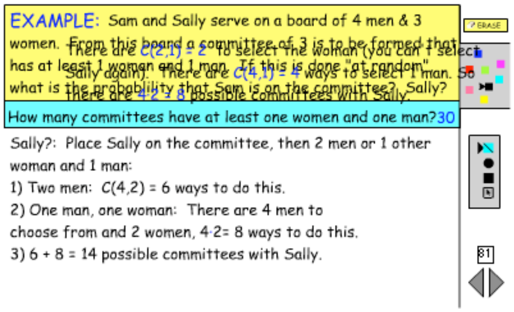
EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

Sally?: Place Sally on the committee, then 2 men or 1 other woman and 1 man:

1) Two men: $C(4,2) = 6$ ways to do this.
2) One man, one woman: There are 4 men to choose from and 2 women, $4 \cdot 2 = 8$ ways to do this.

There are $C(2,1) = 2$ to select the woman (you can't select Sally again). There are $C(4,1) = 4$ ways to select 1 man. So there are $4 \cdot 2 = 8$ possible committees with Sally.



EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

There are $C(2,1) = 2$ to select the woman (you can't select Sally again). There are $C(4,1) = 4$ ways to select 1 man. So there are $4 \cdot 2 = 8$ possible committees with Sally.

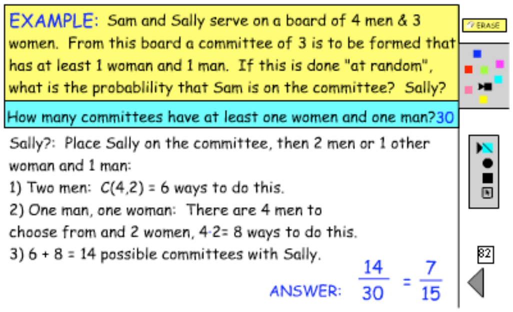
How many committees have at least one women and one man? 30

Sally?: Place Sally on the committee, then 2 men or 1 other woman and 1 man:

1) Two men: $C(4,2) = 6$ ways to do this.

2) One man, one woman: There are 4 men to choose from and 2 women, $4 \cdot 2 = 8$ ways to do this.

3) $6 + 8 = 14$ possible committees with Sally.



EXAMPLE: Sam and Sally serve on a board of 4 men & 3 women. From this board a committee of 3 is to be formed that has at least 1 woman and 1 man. If this is done "at random", what is the probablility that Sam is on the committee? Sally?

How many committees have at least one women and one man? 30

Sally?: Place Sally on the committee, then 2 men or 1 other woman and 1 man:

1) Two men: $C(4,2) = 6$ ways to do this.

2) One man, one woman: There are 4 men to choose from and 2 women, $4 \cdot 2 = 8$ ways to do this.

3) $6 + 8 = 14$ possible committees with Sally.

ANSWER: $\frac{14}{30} = \frac{7}{15}$

Lecture 7