

PERMUTATIONS:

DEFINITION: A permutation is an ordered list of elements from a set (without repetition).

Example: Let $S = \{4,2,3,1\}$. Then $(3,1,2)$ is a permutation - meaning that 3 is first on the list, 1 is second, and 2 is third. This would be called a permutation of length 3.

Very often, the list will represent a selection process. In this case, it could have happened that 3 was chosen first, then 1, then 2.

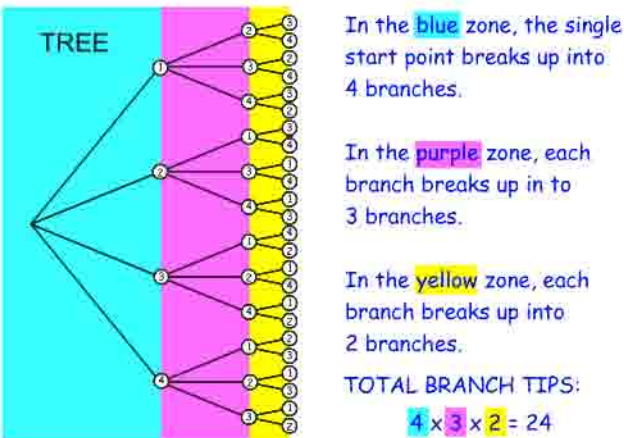
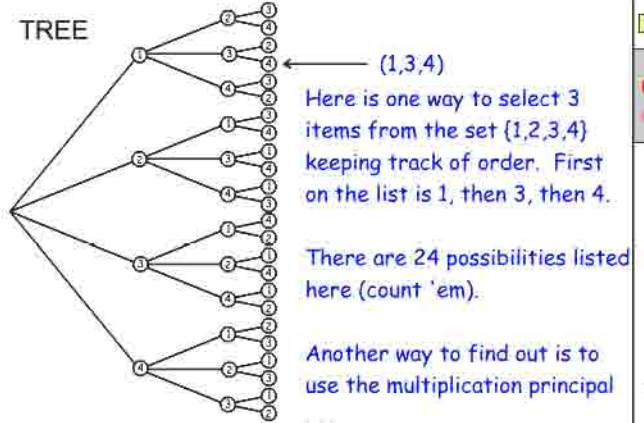
MAJOR QUESTION: How many permutation are there of a given length (taken from a given set).

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SOLUTION: One way to do this is to draw a tree, and count the number of outcomes. Another way is to use the multiplication principal. At each stage in the tree, the branches split up into the same number of new branches



EXAMPLE: Let $S = \{1,2,3,4\}$. How many permutations are there of length 3?

Perhaps the easiest way to approach this problem is to use the multiplication principal. You have to fill 3 positions using the symbols 1, 2, 3, 4 (with no repetition).

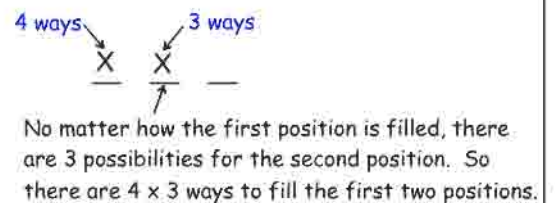
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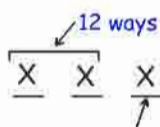
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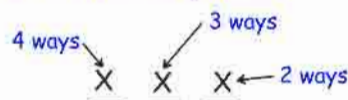


No matter how the first two positions are filled, there are 2 ways to fill in the last position (because there are two numbers left).

ERASE

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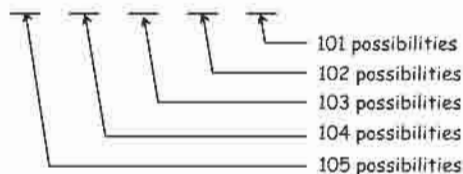
Answer: $4 \times 3 \times 2 = 24$.

ERASE

Lecture 5

EXAMPLE: Let S be a set with 105 elements. How many ways can you select 5 items from the set keeping track of order?

Fill 5 positions:

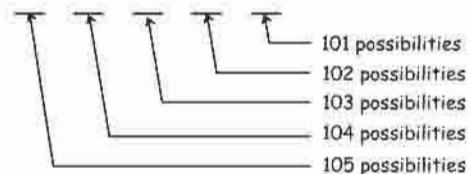


ERASE

EXAMPLE: Let S be a set with 105 elements. How many ways can you select 5 items from the set keeping track of order?

Fill 5 positions:

ANSWER: $105 \times 104 \times 103 \times 102 \times 101$



ERASE

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Fill 5 positions:

ANSWER: $105 \times 104 \times 103 \times 102 \times 101$

— — — — —

You are invited to draw the tree to verify this. Anybody that does this correctly gets -1 points added to their midterm score.

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EXAMPLE: Let S be a set with n elements. How many ways can you select 1 items from the set keeping track of order?

Fill 1 positions:

—

ERASE

EXAMPLE: Let S be a set with n elements. How many ways can you select 1 items from the set keeping track of order?

Fill 1 positions:

Answer: n



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EXAMPLE: Let S be a set with n elements. How many ways can you select 2 items from the set keeping track of order?

Fill 2 positions:

Answer: $n(n-1)$
 $= n(n-2+1)$

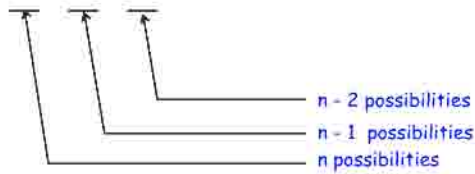


ERASE

EXAMPLE: Let S be a set with n elements. How many ways can you select 3 items from the set keeping track of order?

Fill 3 positions:

$$\text{Answer: } n(n-1)(n-2) \\ = n(n-1)(n-3+1)$$



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17

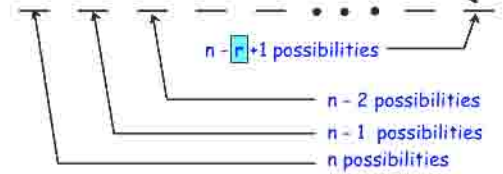


Lecture 5

EXAMPLE: Let S be a set with n elements. How many ways can you select r items from the set keeping track of order?

Fill r positions:

$$\text{ANSWER: } n(n-1)(n-2) \cdots (n-r+1)$$



ERASE



18



EXAMPLE: Let S be a set with n elements. How many ways can you select r items from the set keeping track of order?

$$\text{ANSWER: } n(n-1)(n-2) \cdots (n-r+1)$$

There is notation for this (as always).

$P(n,r) = n(n-1)(n-2) \cdots (n-r+1)$ = the number of ways to select r objects from a set of n objects keeping track of order.

ERASE



19



SOME NOTATION:

$$2! = 2 \cdot 1$$

$$3! = 3 \cdot 2 \cdot 1$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

...

$$n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$$

ERASE



20



SOME NOTATION:

$$0! = 1$$

$$1! = 1$$

$$2! = 2 \cdot 1$$

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ERASE



21



$P(n,r) = n(n-1)(n-2) \cdots (n-r+1)$ = the number of ways to select r objects from a set of n objects keeping track of order.

$$P(n,r) = n(n-1)(n-2) \cdots (n-r+1) =$$

$$= \frac{n(n-1)(n-2) \cdots (n-r+1) \cdot \underbrace{(n-r)(n-r-1) \cdots 3 \cdot 2 \cdot 1}_{(n-r)!}}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

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22



$P(n,r) = n(n-1)(n-2) \cdots (n-r+1)$ = the number of ways to select r objects from a set of n objects keeping track of order.

$$P(n,r) = \frac{n!}{(n-r)!} = n(n-1)(n-2) \cdots (n-r+1)$$

NOTICE:

$$P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

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23



EXAMPLE: Let $S = \{1,2,3,4,5,7,9\}$. Form a four digit number by selecting four of the elements in S without replacement. How many numbers are possible?

SOLUTION: As before, 4 slots are going to be filled.

— — — —

This is just a matter of writing down $P(7,4) = 7 \cdot 6 \cdot 5 \cdot 4 = 840$.

ERASE



24



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CONTINUATION: How many numbers are possible that contain both even and odd digits?

ERASE



25



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$$840 = \#(\text{all evens}) + \#(\text{both evens and odds}) + \#(\text{all odds})$$

ERASE



26



Lecture 5 ■■

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This is what we want.

ERASE



27



EXAMPLE: Let $S = \{1,2,3,4,5,7,9\}$.

$\#(\text{all evens}) = 0$ There are only two evens in S .

$\#(\text{all odds}) = ?$

CONTINUATION: How many numbers are possible that contain both even and odd digits?

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28



EXAMPLE: Let $S = \{1,2,3,4,5,7,9\}$.

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29



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$$\#(\text{all odds}) = P(5,4) = 5 \cdot 4 \cdot 3 \cdot 2 = 120$$

CONTINUATION: How many numbers are possible that contain both even and odd digits?

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This is what we want.

ERASE



30



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$$840 = 0 + \#(\text{both evens and odds}) + 120$$

ERASE



31



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$$\#(\text{all odds}) = P(5,4) = 5 \cdot 4 \cdot 3 \cdot 2 = 120$$

CONTINUATION: How many numbers are possible that contain both even and odd digits?

$$840 = \#(\text{all evens}) + \#(\text{both evens and odds}) + \#(\text{all odds})$$

$$\#(\text{both evens and odds}) = 840 - 120 = 720$$

ERASE



32



EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

ERASE



33



Lecture 5 ■ ■

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34



SOLUTION: How many ways can the female roles be filled?

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35



SOLUTION: How many ways can the female roles be filled?

Notice that order is imposed by the fact that the roles are distinct (for no other reason that the people must appear at different positions on stage). It may as well be assumed that there is a first female role, a second, third, and fourth female role. So assigning the four roles is the same as selecting 4 of the females **KEEPING TRACK OF ORDER**. (Assign the first choice to role 1, the second choice to role 2 etc.) This is a problem about permutations.

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36



SOLUTION: How many ways can the female roles be filled?

From a set of seven people, 4 are selected with regard to order. There are $P(7,4) = 7 \cdot 6 \cdot 5 \cdot 4 = 840$ ways to do this.

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ERASE



38



SOLUTION: How many ways can the female roles be filled?

From a set of seven people, 4 are selected with regard to order. There are $P(7,4) = 7 \cdot 6 \cdot 5 \cdot 4 = 840$ ways to do this.

How many ways can the male roles be filled?

From a set of eight people, 2 are selected with regard to order. There are $P(8,2) = 8 \cdot 7 = 56$ ways to do this.

EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

ERASE



39



SOLUTION: How many ways can the female roles be filled?

840

How many ways can the male roles be filled?

56

EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

ERASE



40



How many ways can the female roles be filled?

840

How many ways can the male roles be filled?

56

EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

QUICK WAY:

$$\frac{7}{F} \frac{6}{F} \frac{5}{F} \frac{4}{F} \frac{8}{M} \frac{7}{M}$$

- 7 ways to fill the first role
- 6 ways to fill the second role for EACH of the 7 previous choices
- 5 ways to fill the third role for EACH of the 7·6 previous choices
- 4 ways to fill the fourth role for EACH of the 7·6·5 previous choices
- 8 ways to fill the fifth role for EACH of the 7·6·5·4 previous choices
- 7 ways to fill the sixth role for EACH of the 7·6·5·4·8 previous choices

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ANSWER: 7·6·5·4·8·7

Lecture 5 ■■

EXAMPLE: How many 4 letter "words" can be formed using the letters GOOD?

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PARTIAL ANSWER: Pretend that the letters all distinct:
G O₁ O₂ D

This is now a pretty standard problem. 4 places

— — — —

are to be filled with four items, thus forming an ordered list of the items G, O₁, O₂, and D.

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PARTIAL ANSWER: Pretend that the letters are all distinct - G O₁ O₂ D:

This is now a pretty standard problem. 4 places

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are to be filled with four items, thus forming an ordered list of the items G, O₁, O₂, and D. There are 4·3·2·1 = 24 ways to do this.

EXAMPLE: How many 4 letter "words" can be formed using the letters GOOD?

PARTIAL ANSWER: HERE THEY ARE:

GO ₁ O ₂ D	O ₁ GO ₂ D	O ₁ O ₂ GD	O ₁ O ₂ DG
GO ₂ O ₁ D	O ₂ GO ₁ D	O ₂ O ₁ GD	O ₂ O ₁ DG
GO ₁ DO ₂	O ₁ GDO ₂	O ₁ DGO ₂	O ₁ DO ₂ G
GO ₂ DO ₁	O ₂ GDO ₁	O ₂ DGO ₁	O ₂ DO ₁ G
GDO ₁ O ₂	DGO ₁ O ₂	DO ₁ GO ₂	DO ₁ O ₂ G
GDO ₂ O ₁	DGO ₂ O ₁	DO ₂ GO ₁	DO ₂ O ₁ G

Notice: If the 1's and 2's are erased, every "word" appears twice, because every word appears with O₁O₂ and O₂O₁.

EXAMPLE: How many 4 letter "words" can be formed using the letters GOOD?

PARTIAL ANSWER: HERE THEY ARE:

GO O D	O GO D	O O GD	O O DG
GO O D	O GO D	O O GD	O O DG
GO DO	O GDO	O DGO	O DO G
GO DO	O GDO	O DGO	O DO G
GDO O	DGO O	DO GO	DO O G
GDO O	DGO O	DO GO	DO O G

Notice: If the 1's and 2's are erased, every "word" appears twice, because every word appears with O₁O₂ and O₂O₁.

ANSWER: 24/2 = 12

EXAMPLE: How many 4 letter "words" can be formed using the letters GOOD?

This last bit of reasoning could have been done without writing out all the permutations of G, O₁, O₂, and D.

Notice: If the 1's and 2's are erased, every "word" appears twice, because every word appears with O₁O₂ and O₂O₁.

ANSWER: 24/2 = 12

EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

How many ways can the female roles be filled? 840

How many ways can the male roles be filled? 56

ERASE



41



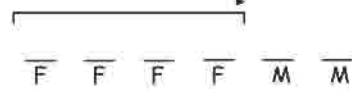
Lecture 5 ■■

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So, to fill all of the roles there are 840 ways to fill the female roles:



ERASE



42

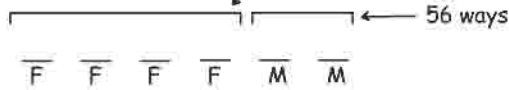


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For each one of these 840 ways, there are 56 ways to proceed and fill the male roles.

ERASE



43

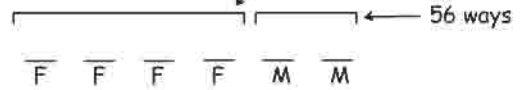


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How many ways can the male roles be filled? 56

So, to fill all of the roles there are 840 ways to fill the female roles:



For each one of these 840 ways, there are 56 ways to proceed and fill the male roles. **ANSWER: 840 x 56 = 47,040**

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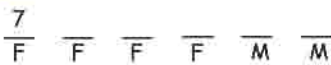


44



EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

QUICK WAY:



• 7 ways to fill the first role

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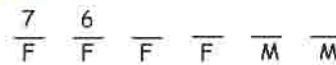


45



EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

QUICK WAY:



• 7 ways to fill the first role

• 6 ways to fill the second role for EACH of the 7 previous choices

ERASE

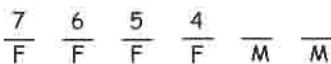


46



EXAMPLE: 7 females and 8 males try out for a play. 4 female roles need to be filled and 2 male roles need to be filled. How many can this be done? (Assume that there are no cross-dressers.)

QUICK WAY:



• 7 ways to fill the first role

• 6 ways to fill the second role for EACH of the 7 previous choices

• 5 ways to fill the third role for EACH of the 7·6 previous choices

• 4 ways to fill the fourth role for EACH of the 7·6·5 previous choices

ERASE

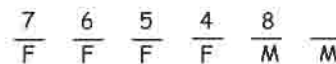


47



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• 8 ways to fill the fifth role for EACH of the 7·6·5·4 previous choices

ERASE



48



EXAMPLE: How many 4 letter "words" can be formed using the letters GOOD?

There are $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways to arrange the letters G, O₁, O₂, and D to form a 4 letter word. Half of these words will look like $\dots O_1 \dots O_2 \dots$ and half will look like $\dots O_2 \dots O_1 \dots$.

$$\text{ANSWER: } \frac{4 \cdot 3 \cdot 2 \cdot 1}{2} = \frac{4!}{2!} = 12$$

ERASE



57



Lecture 5

EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

ERASE



58



EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

First, what is the answer if the letters are distinct?

$$P_1 E_1 E_2 P_2 E_3 R$$

ANSWER: $P(6,6) = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

ERASE



59



EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

First, what is the answer if the letters are distinct?

$$P_1 E_1 E_2 P_2 E_3 R$$

ANSWER: $P(6,6) = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$. Suppose a list is made and one word on the list is $E_1 R P_1 E_2 P_2 E_3$. When all of the 1's, 2's, and 3's are erased, how many times will the word ERPEPE appear on the list? All words of the form E.RP.E.P.E. will end up looking like ERPEPE when the 1's, 2's, and 3's are erased.

ERASE



60



EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

E.RP.E.P.E. The letters E₁, E₂, E₃ have to be placed in the 3 "E" positions. There are 3! ways to do this (3 items are placed keeping track of order). For each one of these ways, there will be 2! ways to place the 1, and 2 subscripts on the P's.

CONCLUSION: The E's can be arranged 3! ways and the P's can be arranged 2! ways. The word ERPEPE will appear $3! \cdot 2! = 6 \times 2 = 12$ times on the list, as will every word.

ERASE



61



EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

Here are the 12 ways E.RP.E.P.E. appears on the list.

$E_1 R P_1 E_2 P_2 E_3$	$E_1 R P_1 E_3 P_2 E_2$	$E_2 R P_1 E_1 P_2 E_3$
$E_2 R P_1 E_3 P_2 E_1$	$E_3 R P_1 E_1 P_2 E_2$	$E_3 R P_1 E_2 P_2 E_1$
$E_1 R P_2 E_2 P_1 E_3$	$E_1 R P_2 E_3 P_1 E_2$	$E_2 R P_2 E_1 P_1 E_3$
$E_2 R P_2 E_3 P_1 E_1$	$E_3 R P_2 E_1 P_1 E_2$	$E_3 R P_2 E_2 P_1 E_1$

ERASE



62



EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

ANSWER:

$$\frac{6!}{2!3!}$$

Number of words on list using 6 different letters.

Number of times each word will appear on the list after erasing subscripts.

ERASE



63



EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

ANSWER:

$$\frac{6!}{2!3!}$$

Number of ways to arrange 6 different letters.

Number of ways to arrange the 3 E's.

Number of ways to arrange the 2 P's.

ERASE



64



EXAMPLE: How many 6 letter "words" can be formed using the letters PEEPER?

ANSWER:

$$\frac{6!}{2!3!} = \frac{720}{2 \cdot 6} = 60$$

ERASE



65



Lecture 5 ■ ■