SETS OF OUTCOMES AND TREES

A sample space is a set consisting ofthe outcomes of some procedure. It will serve as the universal set when trying to solve probability problems.

The book refers to "experiments" rather than procedures.

The best way to show what these are and how they are set up is by example:

1

EXAMPLE: Two batteries are tested, one after the other, and the results are recorded as to whether or not they are charged or not charged.

Here is one way to write down the sample space:

$$S = \{CC, CN, NC, NN\}$$

where it is understood (because we are now agreeing to do this) that

CC - represents the outcome in which the first battery tested was charged and so was the second battery tested.

CN - first charged, second not charged

2

Lecture 3

EXAMPLE: Two batteries are tested, one after the other, and the results are recorded as to whether or not they are charged or not charged.

Here is one way to write down the sample space:

$$S = \{CC, CN, NC, NN\}$$

where it is understood (because we are now agreeing to do this) that

NC - first not charged, second charged

NN - first not charged, second not charged

3

EXAMPLE: Two batteries are tested, one after the other, and the results are recorded as to whether or not they are charged or not charged.

Here is one way to write down the sample space:

$$S = \{CC, CN, NC, NN\}$$

This notation is by our own convention and has to be explained as partof the answer.

The outcomes CC CN NC and NN are elements of the set S, our sample space.

Generally, the outcomes will record somehow only the information needed to solve a problem.

4

EXAMPLE: Two batteries are tested, one after the other, and the results are recorded as to whether or not they are charged or not charged.

Here is another way to write down the sample space:

Let

$$B = \{C,N\}$$

where C represents a charged battery and N represents a battery that is not charged.

Set

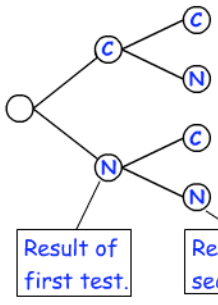
$$S = B \times B = \{(C,C),(C,N),(N,C),(N,N)\}$$

These is a set of ordered pairs. (C,N) has obvious interpretation - first battery charged, second not charged.

5

EXAMPLE: Two batteries are tested, one after the other, as to whether or not they are charged or not charged.

Either way the sample space can be represented by a TREE:



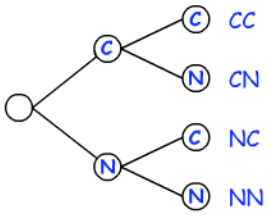
$\Rightarrow$ NN

6

EXAMPLE: Two batteries are tested, one after the other, as to whether or not they are charged or not charged.

Either way the sample space can be represented by a TREE:

BASE or START POINT



CC
CN
NC
NN

The outcomes can be seen on the tips of the branches, and record the results (or inter-mediate outcomes) seen as you travel out from the base to the tips.

7

NEXT EXAMPLE: A coin is flipped 3 times. The result of each toss (heads or tails) is recorded (in order).

Let

$$R = \{H,T\}$$

and let

$$S = R \times R \times R =$$

$$\{(H,H,H), (H,H,T), (H,T,H), (T,H,H), (H,T,T), (T,H,T), (T,T,H), (T,T,T)\}$$

This has obvious interpretation. For example:

8

NEXT EXAMPLE: A coin is flipped 3 times. The result of each toss (heads or tails) is recorded (in order).

Let

$R = \{H,T\}$

and let

$S = RxRxR =$

$\{(H,H,H), (H,H,T), (H,T,H), (T,H,H), (H,T,T), (T,H,T), (T,T,H), (T,T,T)\}$

This represents the outcome of having flipped a heads, then a tails, then a heads.

S is the sample space.

9

EXAMPLE: There are two bowls - bowl A and bowl B. In bowl A is a red ball, a white ball, and a green ball. In bowl of B is a red and a white ball.

A B

A procedure consists of first picking a bowl A or B, recording the pick, and then picking a ball from the chosen bowl and recording the color. Write down the appropriate sample space and corresponding tree.

10

Lecture 3

EXAMPLE:

A B

A procedure consists of first picking a bowl A or B, recording the pick, and then picking a ball from the chosen bowl and recording the color. Write down the appropriate sample space and corresponding tree.

11

EXAMPLE:

A B

A procedure consists of first picking a bowl A or B, recording the pick, and then picking a ball from the chosen bowl and recording the color. Write down the appropriate sample space and corresponding tree.

$S = \{AW, AG, AR, BW, BR\}$

where, for example, AW represents the outcome of picking bowl A and then the white ball.

12

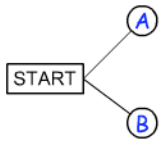
EXAMPLE:

A B

A procedure consists of first picking a bowl A or B, recording the pick, and then picking a ball from the chosen bowl and recording the color. Write down the appropriate sample space and corresponding tree.

First stage:

Two possibilities either pick bowl A or bowl B.



13

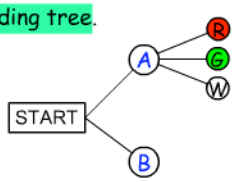
EXAMPLE:

A B

A procedure consists of first picking a bowl A or B, recording the pick, and then picking a ball from the chosen bowl and recording the color. Write down the appropriate sample space and corresponding tree.

Second stage:

3 possibilities if bowl A was selected, and . . .



14

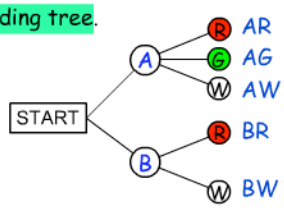
EXAMPLE:

A B

A procedure consists of first picking a bowl A or B, recording the pick, and then picking a ball from the chosen bowl and recording the color. Write down the appropriate sample space and corresponding tree.

Second stage:

two possibilites if bowl B was selected.

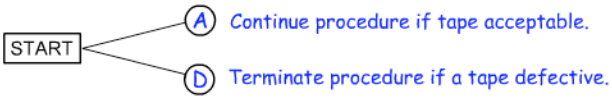


15

EXAMPLE: NOT ALL TREE BRANCHES REACH THE TOP.
Three VCR tapes are checked one after another until either they have all been tested or one defective tape has been found. Draw a tree and then specify the sample space.

16

EXAMPLE: NOT ALL TREE BRANCHES REACH THE TOP.
Three VCR tapes are checked one after another until either they have all been tested or one defective tape has been found. Draw a tree and then specify the sample space.



Stage one:
Test tape 1

17

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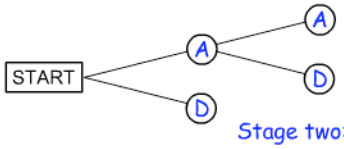


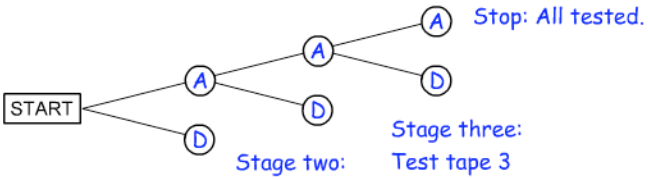
Stage one:
Test tape 1

Stage two:
Test tape 2

18

Lecture 3

EXAMPLE: NOT ALL TREE BRANCHES REACH THE TOP.
Three VCR tapes are checked one after another until either they have all been tested or one defective tape has been found. Draw a tree and then specify the sample space.



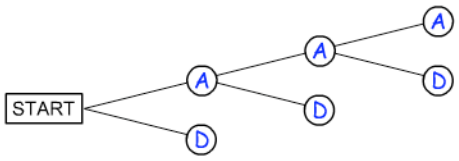
Stage one:
Test tape 1

Stage two:
Test tape 2

Stage three:
Test tape 3

19

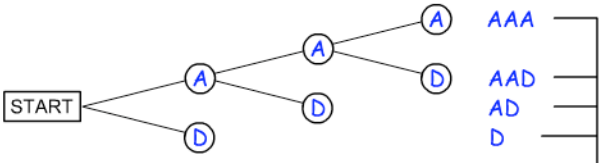
EXAMPLE: NOT ALL TREE BRANCHES REACH THE TOP.
Three VCR tapes are checked one after another until either they have all been tested or one defective tape has been found. Draw a tree and then specify the sample space.



THIS IS THE TREE

20

EXAMPLE: NOT ALL TREE BRANCHES REACH THE TOP.
Three VCR tapes are checked one after another until either they have all been tested or one defective tape has been found. Draw a tree and then specify the sample space.



THIS IS THE TREE

THESE ARE
THE OUTCOMES.

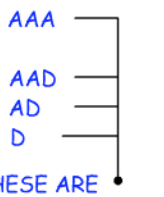
21

EXAMPLE: NOT ALL TREE BRANCHES REACH THE TOP.
Three VCR tapes are checked one after another until either they have all been tested or one defective tape has been found. Draw a tree and then specify the sample space.

The sample space is:

S = {AAA, AAD, AD, D}

where, for example, AAD represents the outcome of first getting two acceptable tapes then a defective.



THESE ARE
THE OUTCOMES.

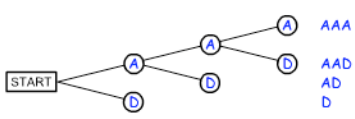
22

EXAMPLE: NOT ALL TREE BRANCHES REACH THE TOP.
Three VCR tapes are checked one after another until either they have all been tested or one defective tape has been found. Draw a tree and then specify the sample space.

FINAL SOLUTION

The sample space is:

S = {AAA, AAD, AD, D}



TREE

where, for example, AAD represents the outcome of first getting two acceptable tapes then a defective.

23

MULTIPLICATION PRINCIPLE:

EXAMPLE: You are going to order a sandwich. First you have to choose beef, ham, or turkey. Next you have to choose rye or white for the bread. How many elements are in the sample space?

SOLUTION: One way to do the problem is to write out the sample space, and count:

S = {BW, BR, HW, HR, TW, TR}
ham on white

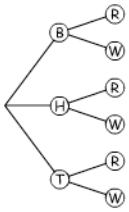
ANSWER: 6

24

MULTIPLICATION PRINCIPLE:

EXAMPLE: You are going to order a sandwich. First you have to choose beef, ham, or turkey. Next you have to choose rye or white for the bread. How many elements are in the sample space?

SOLUTION: Another way is to draw a tree and count:



25

MULTIPLICATION PRINCIPLE:

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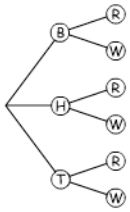


However, notice that there are 3 choices in the first stage, and for EACH one of those there are 2 choices in the second stage.

26

Lecture 3

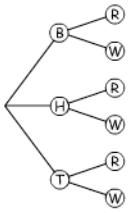
MULTIPLICATION PRINCIPLE:

EACH OF THE FIRST CHOICES "EXPLODES" INTO 2 PATHS

3 choices first stage — 2 choices second stage

$3 \times 2 = 6$

SOLUTION: Another way is to draw a tree and count:



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27

MULTIPLICATION PRINCIPLE:

EACH OF THE FIRST CHOICES "EXPLODES" INTO 2 PATHS

3 choices first stage — 2 choices second stage

$3 \times 2 = 6$

OFTEN IT IS USEFUL TO USE THIS LINE OF REASONING WITHOUT DRAWING THE TREE OR ONLY DRAWING PART OF IT.

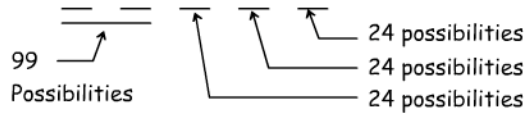
28

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or an O. How many passwords are possible? Example: 34ZED

29

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or an O. How many passwords are possible? Example: 34ZED

SOLUTION: There are 24 possibilities for each letter and 99 possibilities for the number.



30

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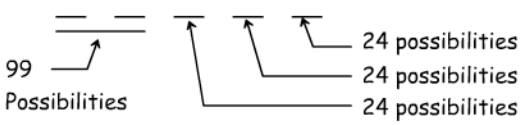


ANSWER: 99x24x24x24

31

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or an O. How many passwords are possible? Example: 34ZED

SOLUTION: What about the tree? It's messy.

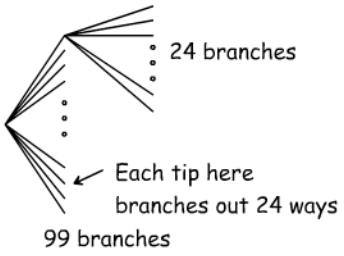
First stage:

99 branches

32

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or an O. How many passwords are possible? Example: 34ZED

SOLUTION: What about the tree? It's messy.



33

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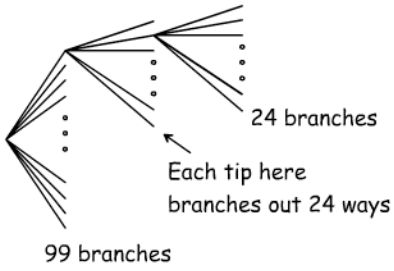

34

Lecture 3

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or an O. How many passwords are possible? Example: 34ZED

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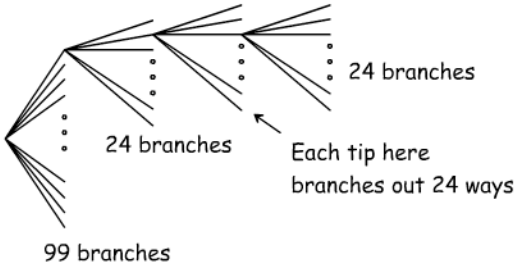

35

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or an O. How many passwords are possible? Example: 34ZED

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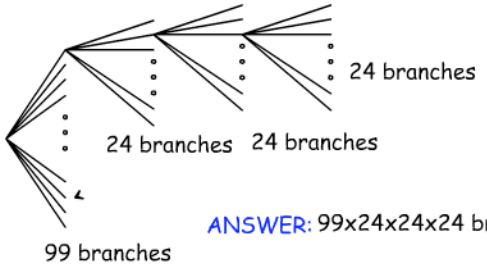


ANSWER: 99x24x24x24 branch tips

36

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or a O. How many passwords do not include the letter Q?

SOLUTION:

37

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or a O. How many passwords do not include the letter Q?

SOLUTION:

There are 23 letters excluding I, O, and Q.

99x23x23x23

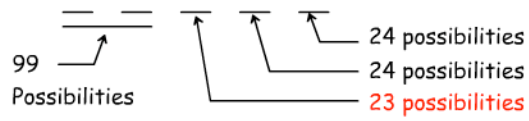
38

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or a O. How many passwords are there where the first letter is not a Q?

39

EXAMPLE: A password consists of a number 1 through 99, followed by three letters. None of the letters is to be an I or a O. How many passwords are there where the first letter is not a Q?

SOLUTION:



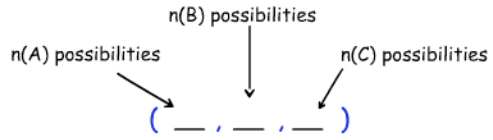
ANSWER: 99x23x24x24

40

CROSS PRODUCTS: How many elements are in $AxBxC$? $AxBxC = \{(a,b,c): a \in A, b \in B, c \in C\}$. To chose an element from this set, you have to fill in 3 "blanks" to make up the ordered triples that are in $AxBxC$.

(__ , __ , __)

CROSS PRODUCTS: How many elements are in $AxBxC$? $AxBxC = \{(a,b,c): a \in A, b \in B, c \in C\}$. To chose an element from this set, you have to fill in 3 "blanks" to make up the ordered triples that are in $AxBxC$.

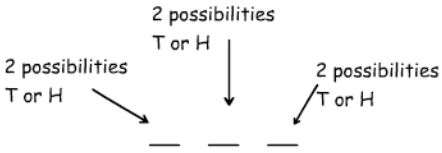


$n(A) \times n(B) \times n(C) = n(AxBxC)$

MORE EXAMPLES:

EXAMPLE: Flip a coin 3 times. Record the results (heads or tails) in the order that they occur. How many elements are in the sample space?

SOLUTION: A typical element in the sample space will look like TTH. representing a tails then a tails then a heads. In general 3 positions have to be filled with T or H.



2x2x2 = 8