Lecture 29

1

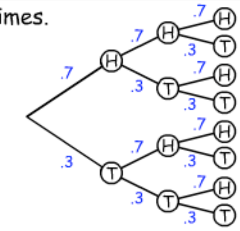
STOCHASTIC PROCESS: A stochastic process is a process in which one thing happens after another. Example: Anything involving a tree. There is a first step, then a second, . . .

2

STOCHASTIC PROCESS: A stochastic process is a process in which one thing happens after another. Example: Anything involving a tree. There is a first step, then a second, . . .

BERNOULLI PROCESS: This is a stochastic process in which every step has the same possibilities, and these possibilities have the same probabilities of occurence on every step.

Example: Toss an unfair coin 3 times.



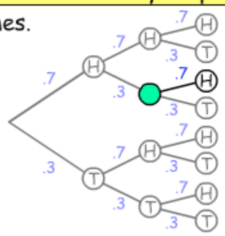
3

STOCHASTIC PROCESS: A stochastic process is a process in which one thing happens after another. Example: Anything involving a tree. There is a first step, then a second, . . .

BERNOULLI PROCESS: This is a stochastic process in which every step has the same possibilities, and these possibilities have the same probabilities of occurence on every step.

Example: Toss an unfair coin 3 times.

Notice that at every branch point, you see a .7 (H) emanating from it.



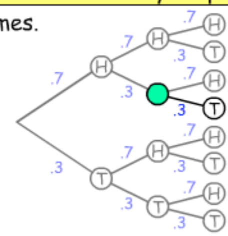
4

STOCHASTIC PROCESS: A stochastic process is a process in which one thing happens after another. Example: Anything involving a tree. There is a first step, then a second, . . .

BERNOULLI PROCESS: This is a stochastic process in which every step has the same possibilities, and these possibilities have the same probabilities of occurence on every step.

Example: Toss an unfair coin 3 times.

Notice that at every branch point, you see a .7 (H) emanating from it. You also see .3 (T) emanating from it.



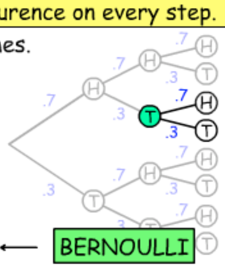
5

STOCHASTIC PROCESS: A stochastic process is a process in which one thing happens after another. Example: Anything involving a tree. There is a first step, then a second, . . .

BERNOULLI PROCESS: This is a stochastic process in which every step has the same possibilities, and these possibilities have the same probabilities of occurence on every step.

Example: Toss an unfair coin 3 times.

Notice that at every branch point, you see a .7 (H) emanating from it. You also see .3 (T) emanating from it.



Same possibilities (H or T), same probabilities (.7 and .3). ← BERNOULLI

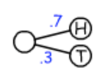
6

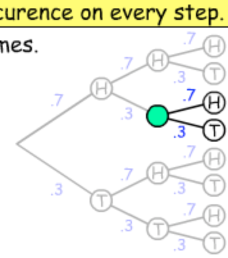
STOCHASTIC PROCESS: A stochastic process is a process in which one thing happens after another. Example: Anything involving a tree. There is a first step, then a second, . . .

BERNOULLI PROCESS: This is a stochastic process in which every step has the same possibilities, and these possibilities have the same probabilities of occurence on every step.

Example: Toss an unfair coin 3 times.

EVERY branch looks like:





7

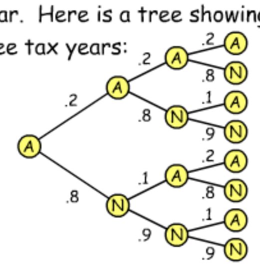
STOCHASTIC PROCESS: A stochastic process is a process in which one thing happens after another. Example: Anything involving a tree. There is a first step, then a second, . . .

A Markov process is a stochastic process that is slightly more general than a Bernoulli process, but still has a uniform pattern to its probabilities. Here is an example:

8

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

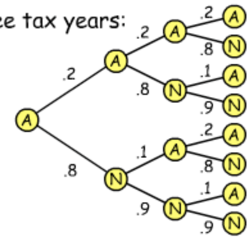
Suppose you were audited this year. Here is a tree showing the possibilities for the next three tax years:



EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

Suppose you were audited this year. Here is a tree showing the possibilities for the next three tax years:

What is the probability, that you will be audited 3 years later?



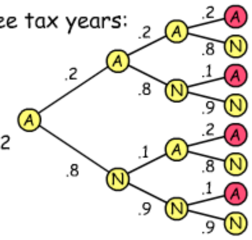
9

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

Suppose you were audited this year. Here is a tree showing the possibilities for the next three tax years:

What is the probability, that you will be audited 3 years later?

$.2 \cdot .2 \cdot .2 + .2 \cdot .8 \cdot .1 + .8 \cdot .1 \cdot .2 + .8 \cdot .9 \cdot .1 = .112$



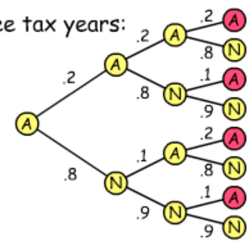
10

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

Suppose you were audited this year. Here is a tree showing the possibilities for the next three tax years:

What is the probability, that you will be audited 10 years later?

This turns out to be not that hard. But the answer will come much later.

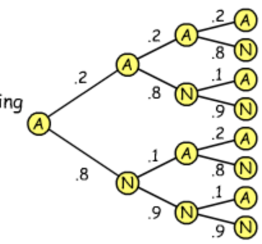


11

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

In any given year, you are in one of two states:

1) State A - the state of being audited that year.
2) State N - the state of not being audited that year.



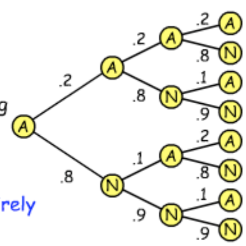
12

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

In any given year, you are in one of two states:

1) State A - the state of being audited that year.
2) State N - the state of not being audited that year.

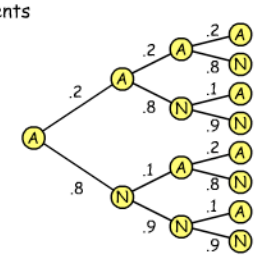
FACT: The probability that you will be in state A next year is determined entirely by your current state.



13

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

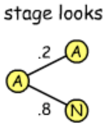
Notice that if a branch point represents state A

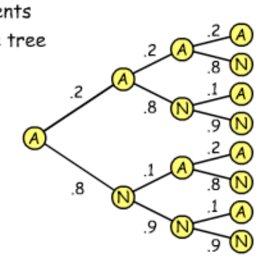


14

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

Notice that if a branch point represents state A, then the continuation of the tree to the next stage looks like:

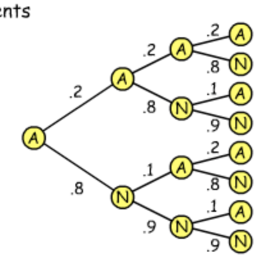




15

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

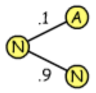
Notice that if a branch point represents state N

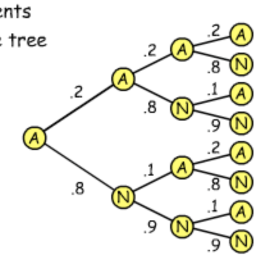


16

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

Notice that if a branch point represents state N, then the continuation of the tree to the next stage looks like:

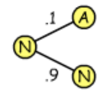


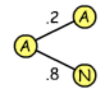


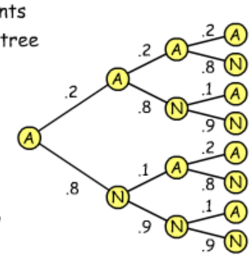
17

EXAMPLE: Every year, the IRS either audits your tax return or does not audit it. If you were NOT audited in a particular year, then the probability of an audit the next year is .1. If you were audited in a particular year, then the probability of and audit the next year is .2.

Notice that if a branch point represents state B, then the continuation of the tree to the next stage looks like:







The probabilities as to "what happens next" depend **only on your current state** (not how you got there or anything else).

18

A Markov chain is a stochastic process for which:

1) At each stage, the result is one of a fixed number of states.

2) At each stage the probability of transition from one state to another depends only on the initial and final states (of that transition).

19

EXAMPLE: A system can be in one of three states - 1, 2, or 3 (e.g. a stop light can be red, green, or yellow). On each transition:

- If it is in state 1, then it has probability 1/2 of ending up in state 2, and probability of 1/2 of staying in state 1.
- If it is in state 2, then it has probability 1 of ending up in state 3.
- If it is in state 3, then it has probability 1/2 of ending up in state 1, and probability of 1/2 of staying in state 3.

20

EXAMPLE: A system can be in one of three states - 1, 2, or 3 (e.g. a stop light can be red, green, or yellow). On each transition:

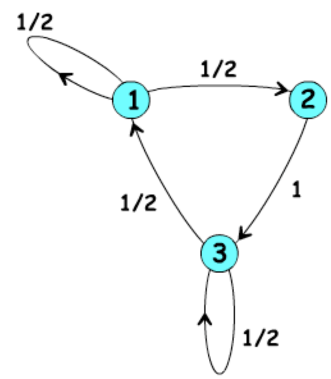
- If it is in state 1, then it has probability 1/2 of ending up in state 2, and probability of 1/2 of staying in state 1.
- If it is in state 2, then it has probability 1 of ending up in state 3.
- If it is in state 3, then it has probability 1/2 of ending up in state 1, and probability of 1/2 of staying in state 3.

ALL OF THIS CAN BE REPRESENTED IN A

TRANSITION DIAGRAM

21

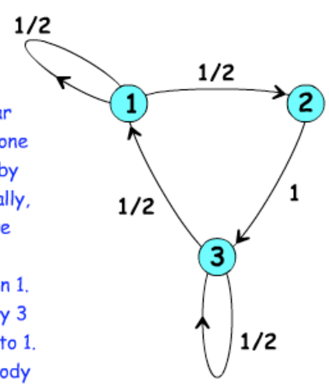
TRANSITION DIAGRAM



22

Here is a way to concretely represent this 3 state system:

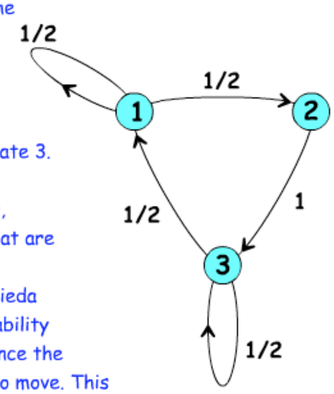
Suppose that states 1, 2, and 3 represent 3 countries. Every year on Jan. 1, the people move from one country to another as indicated by the transition diagram. Specifically, 1/2 of the people in Country 1 are selected at random and sent to Country 2. The other half stay in 1. Also on 1/2 of the people Country 3 are selected at random to move to 1. The other half stay put. Everybody in Country 2 moves to Country 3.



23

There is one special person in the population named Frieda. Whatever country Frieda is in is considered to be the state of the system. If Frieda is in Country 3, the system is in state 3.

Notice that when a move occurs, the probabilities are exactly what are called for in the system. If the system is in state 3 then Frieda is in Country 3. She has a probability of 1/2 or moving to Country 1 since the people are selected at random to move. This agrees with the system transition probabilities.

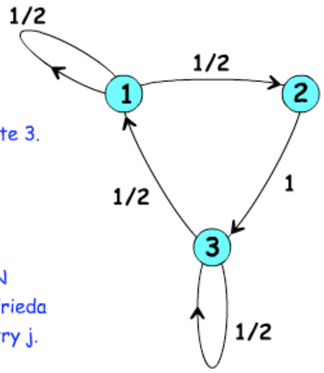


24

There is one special person in the population named Frieda. Whatever country Frieda is in is considered to be the state of the system. If Frieda is in Country 3, the system is in state 3.

STATE OF SYSTEM = Country Frieda is in.

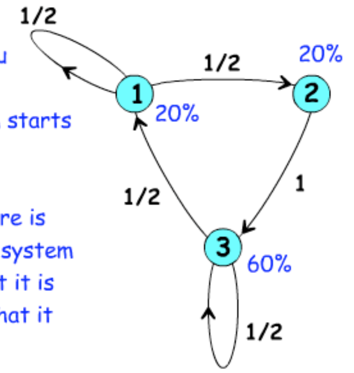
PROBABILITY OF TRANSITION (from state i to j) = Probability Frieda will move from Country i to Country j.



25

One more wrinkle: You may not know the initial state of the system. Perhaps, you only know that there is a probability that the system starts in state 1 or 2 or 3.

Example: Suppose that there is a probability of .2 that the system is in state 1, prob. of .2 that it is in state 2, and prob. of .6 that it is in state 3.

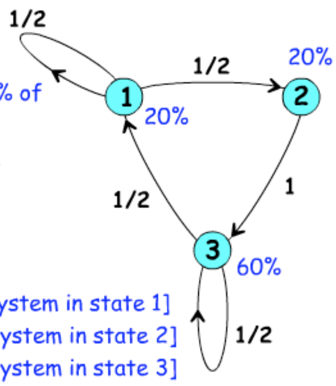


26

How do you represent this using Frieda?

Answer: **Randomly** place 20% of the world's population in Country 1, 20% in Country 2, and 60% in Country 3.

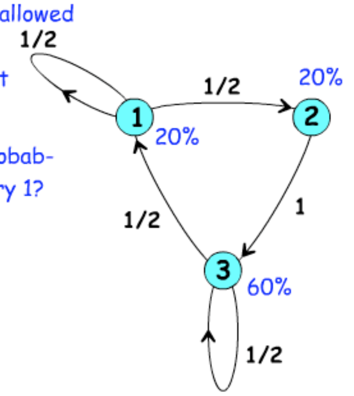
Then
Pr[Frieda is in 1] = .2 = Pr[System in state 1]
Pr[Frieda is in 2] = .2 = Pr[System in state 2]
Pr[Frieda is in 3] = .6 = Pr[System in state 3]



27

Now suppose the system is allowed to under go one transition. What is the probability that it is in state 1?

Equivalently, what is the probability that Frieda is in Country 1?

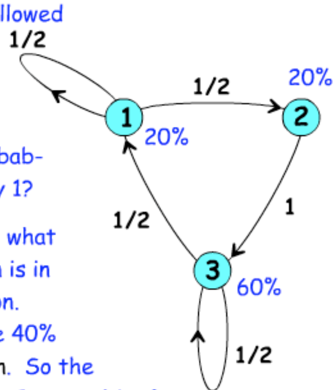


28

Now suppose the system is allowed to under go one transition. What is the probability that it is in state 1?

Equivalently, what is the probability that Frieda is in Country 1?

This is easy - just figure out what percentage of the population is in Country 1 after one transition. Suppose 40% are in 1. Those 40% represent a random selection. So the probability of Frieda being in Country 1 is .4.

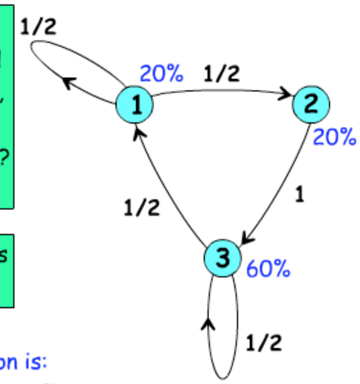


29

QUESTION: After January 1 comes around and the people make their move, what percentage of the population lives in Country 1? Country 2? Country 3?

After one transition, what is the population distribution?

Initial population distribution is:
C1: 20% C2: 20% C3: 60%

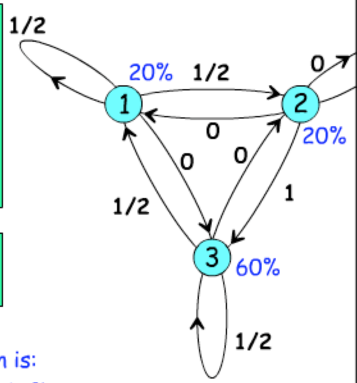


30

QUESTION: After January 1 comes around and the people make their move, what percentage of the population lives in Country 1? Country 2? Country 3?

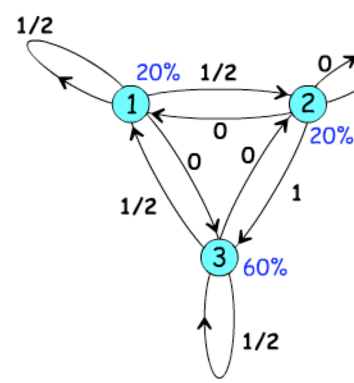
After one transition, what is the population distribution?

Initial population distribution is:
C1: 20% C2: 20% C3: 60%



31

What percentage is in Country 3 after one transition?

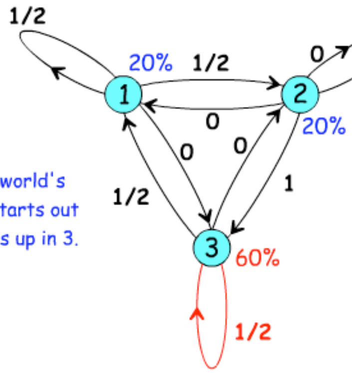


32

What percentage is in Country 3 after one transition?

.6 · 1/2

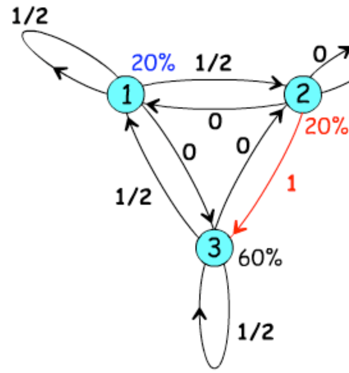
30% of the world's population starts out in 3 and ends up in 3.



What percentage is in Country 3 after one transition?

.2 · 1 + .6 · 1/2

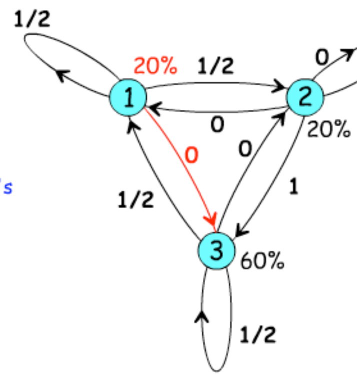
All 20% in 2 move to 3.



What percentage is in Country 3 after one transition?

.2 · 0 + .2 · 1 + .6 · 1/2

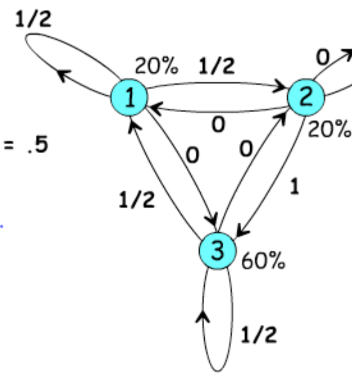
NONE of the 20% of the world's population living in 1 move to 3.



What percentage is in Country 3 after one transition?

.2 · 0 + .2 · 1 + .6 · 1/2 = .5

50% end up in Country 3.

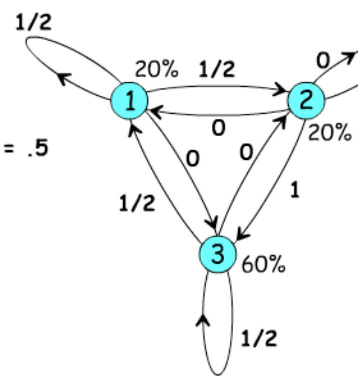


What percentage is in Country 3 after one transition?

.2 · 0 + .2 · 1 + .6 · 1/2 = .5

This can also be written as

$$[.2, .2, .6]\begin{bmatrix}0\\1\\1/2\end{bmatrix} = .5$$



What percentage is in Country 3 after one transition?

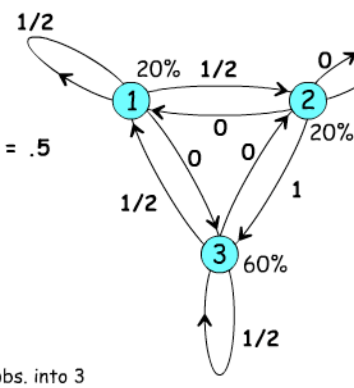
.2 · 0 + .2 · 1 + .6 · 1/2 = .5

This can also be written as

$$[.2, .2, .6]\begin{bmatrix}0\\1\\1/2\end{bmatrix} = .5$$

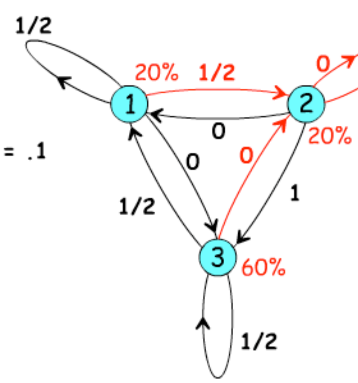
Population distribution prior to transition

transition probs. into 3



What percentage is in Country 2 after one transition?

.2 · 1/2 + .2 · 0 + .6 · 0 = .1



What percentage is in Country 2 after one transition?

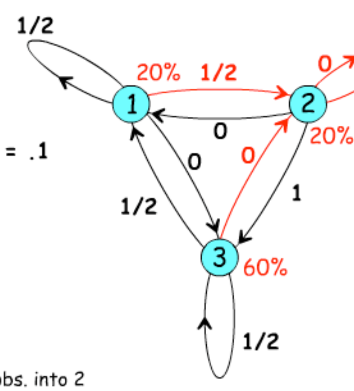
.2 · 1/2 + .2 · 0 + .6 · 0 = .1

This can also be written as

$$[.2, .2, .6]\begin{bmatrix}1/2\\0\\0\end{bmatrix} = .1$$

Population distribution prior to transition

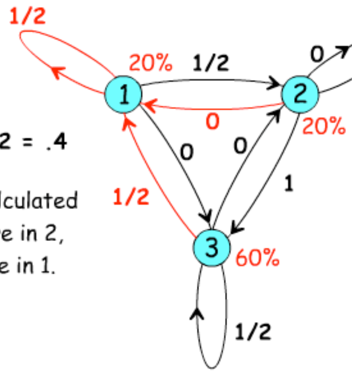
transition probs. into 2



What percentage is in Country 1 after one transition?

$.2 \cdot 1/2 + .2 \cdot 0 + .6 \cdot 1/2 = .4$

Another way: We already calculated that 50% are in 3 and 10% are in 2, so the remaining 40% must be in 1.



What percentage is in Country 1 after one transition?

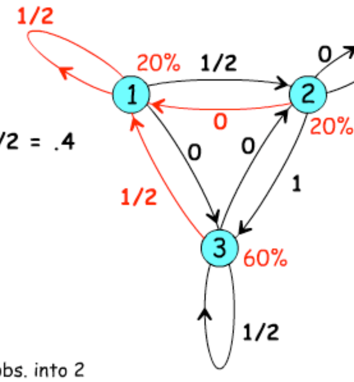
$.2 \cdot 1/2 + .2 \cdot 0 + .6 \cdot 1/2 = .4$

This can also be written as

$$[.2, .2, .6] \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix} = .4$$

Population distribution prior to transition

transition probs. into 2



COUNTRY 1:

$$[.2, .2, .6] \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix} = .4$$

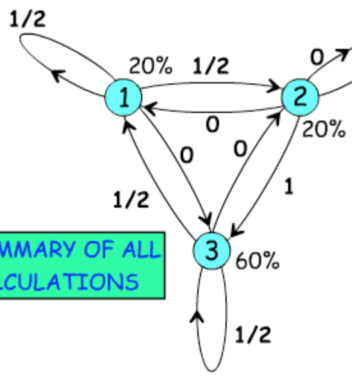
COUNTRY 2:

$$[.2, .2, .6] \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix} = .1$$

COUNTRY 3:

$$[.2, .2, .6] \begin{bmatrix} 0 \\ 1 \\ 1/2 \end{bmatrix} = .5$$

SUMMARY OF ALL CALCULATIONS



$[.2, .2, .6]$ $\begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$ $.4$

$[.2, .2, .6]$ $\begin{matrix} 1/2 \\ 0 \\ 0 \end{matrix}$ $.1$

$[.2, .2, .6]$ $\begin{matrix} 0 \\ 1 \\ 1/2 \end{matrix}$ $.5$

$[.2, .2, .6]$

$\begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$ $.4$

$[.2, .2, .6]$ $\begin{matrix} 1/2 \\ 0 \\ 0 \end{matrix}$ $.1$

$[.2, .2, .6]$ $\begin{matrix} 0 \\ 1 \\ 1/2 \end{matrix}$ $.5$

COUNTRY 1:

$$[.2, .2, .6] \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix} = .4$$

$$[.2, .2, .6] \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix} = [.4, .1, .5]$$

COUNTRY 3:

$$[.2, .2, .6] \begin{bmatrix} 0 \\ 1 \\ 1/2 \end{bmatrix} = .5$$

$$[.2, .2, .6] \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix} = [.4, .1, .5]$$

Population distribution prior to transition — Transition Matrix — Population distribution after transition

QUESTION: What is the population distribution after one more transition?

$$[.2, .2, .6] \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix} = [.4, .1, .5]$$

Population distribution prior to transition — Transition Matrix — Population distribution after transition

QUESTION: What is the population distribution after one more transition?

$$\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix} = [.4, .1, .5]$$

Population distribution prior to transition — Transition Matrix — Population distribution after transition

49

QUESTION: What is the population distribution after one more transition?

$$[.4, .1, .5]\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix} =$$

Population distribution after one transition — Transition Matrix

50

QUESTION: What is the population distribution after one more transition?

$$[.4, .1, .5]\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix} = [.45, .2, .35]$$

Population distribution after one transition — Transition Matrix — Population distribution after two transitions

51

QUESTION: What is the population distribution after one more transition?

$$[.4, .1, .5]\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix} = [.45, .2, .35] =$$

$$[.2, .2, .6]\left[\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix}\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix}\right]$$

52

QUESTION: What is the population distribution after one more transition?

$$[.4, .1, .5]\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix} = [.45, .2, .35] =$$

$$[.2, .2, .6]\left[\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix}\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix}\right]$$

53

QUESTION: What is the population distribution after one more transition?

$$[.4, .1, .5]\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix} = [.45, .2, .35] =$$

$$[.2, .2, .6]\begin{bmatrix}1/4 & 1/4 & 1/2\\ 1/2 & 0 & 1/2\\ 1/2 & 1/4 & 1/4\end{bmatrix}$$

54

QUESTION: What is the population distribution after one more transition?

$$[.4, .1, .5]\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix} = [.45, .2, .35] =$$

$$[.2, .2, .6]\begin{bmatrix}1/4 & 1/4 & 1/2\\ 1/2 & 0 & 1/2\\ 1/2 & 1/4 & 1/4\end{bmatrix} = [.45, .2, .35]$$

55

The multiplication on the right by the transition matrix P gives the new population distribution:

$$\underset{D_1}{[.4, .1, .5]}\ \underset{P}{\begin{bmatrix}1/2 & 1/2 & 0\\ 0 & 0 & 1\\ 1/2 & 0 & 1/2\end{bmatrix}} = \underset{D_2}{[.45, .2, .35]}$$

General Pattern: $D_0 P = D_1$ $\quad D_1 P = D_2$

$(D_0 P)P = D_0 P^2 = D_2$

56

The multiplication on the right by the transition matrix P gives the new population distribution:

$$\underset{D_1}{[.4, .1, .5]}\ \underset{P}{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix}} = \underset{D_2}{[.45, .2, .35]}$$

General Pattern: $D_0\,P = D_1$ $\quad D_1\,P = D_2$

$(D_0\,P)P = D_0\,P^2 = D_2$

$D_0\,P^n = D_n$

57

EXAMPLE: What is the probability distribution after 3 transitions?

$D_0 = [.2, .2, .6]$ $\quad P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$D_0\,P^n = D_n$

58

EXAMPLE: What is the probability distribution after 3 transitions?

$D_0 = [.2, .2, .6]$ $\quad P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$$P^3 = \begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix}$$

$D_0\,P^n = D_n$

59

EXAMPLE: What is the probability distribution after 3 transitions?

$D_0 = [.2, .2, .6]$ $\quad P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$$[.2, .2, .6] \begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix} =$$

$D_0\,P^n = D_n$

60

EXAMPLE: What is the probability distribution after 3 transitions?

$D_0 = [.2, .2, .6]$ $\quad P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$$[.2, .2, .6] \begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix} = [.4, 9/40, 3/8]$$

$D_0\,P^n = D_n$

61

EXAMPLE: What is the equivalent statement in terms of Frieda?

$D_0 = [.2, .2, .6]$ $\quad P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$$[.2, .2, .6] \begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix} = [.4, 9/40, 3/8]$$

[.2, .2, .6]: Initial probability distribution for Frieda. E.g.

Pr[Frieda is in C1] = .2

[.4, 9/40, 3/8]: Probability distribution for Frieda after 3 transitions if she starts with D_0.

Pr[Frieda is in C2 after 3] = 9/40

62

EXAMPLE: What is the equivalent statement in terms of states?

$D_0 = [.2, .2, .6]$ $\quad P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$$[.2, .2, .6] \begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix} = [.4, 9/40, 3/8]$$

[.2, .2, .6]: Initial probability distribution for system. E.g.

Pr[System in state 1] = .2

[.4, 9/40, 3/8]: Probability distribution for system after 3 transitions if it starts with D_0.

Pr[System is in state 2 after 3] = 9/40

63

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

64

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

To answer this, the initial probability distribution is

[1, 0, 0]

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

To answer this, the initial probability distribution is

[1, 0, 0]

Now multiply by P^3 to get the probability distribution after 3 transitions.

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

$$[1, 0, 0]\begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix}$$

Now multiply by P^3 to get the probability distribution after 3 transitions.

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

$$[1, 0, 0]\begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix} = [3/8, 1/8, 1/2]$$

Now multiply by P^3 to get the probability distribution after 3 transitions.

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

$$[1, 0, 0]\begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix} = [3/8, 1/8, 1/2]$$

Now multiply by P^3 to get the probability distribution after 3 transitions.

After 3 transitions,

Pr[System is in state 2 after 3 transitions] = 1/8

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

$$[1, 0, 0]\underbrace{\begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix}}_{P^3} = [3/8, 1/8, 1/2]$$

Notice: This is just the first row of P^3.

$(P^3)_{1,2}$ = probability that the system is in state 2 after 3 transitions if it starts out in state 1.

EXAMPLE: Suppose the system is in state 1. What is the probability that it will be in state 2 after 3 transitions?

$$[1, 0, 0]\underbrace{\begin{bmatrix} 3/8 & 1/8 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 3/8 & 1/4 & 3/8 \end{bmatrix}}_{P^3} = [3/8, 1/8, 1/2]$$

Notice: This is just the first row of P^3.

$(P^3)_{i,j}$ = probability that the system is in state j after 3 transitions if it starts out in state i.

$(P^n)_{i,j}$ = probability that the system is in state j after n transitions if it starts out in state i.

Notation:

$P_{i,j}(n)$ = probability that the system is in state j after n transitions if it starts out in state i.

$(P^n)_{i,j} = P_{i,j}(n)$ = probability that the system is in state j after n transitions if it starts out in state i.

$$P^n = \begin{bmatrix} P_{1,1}(n) & P_{1,2}(n) & P_{1,3}(n) \\ P_{2,1}(n) & P_{2,2}(n) & P_{2,3}(n) \\ P_{3,1}(n) & P_{3,2}(n) & P_{3,3}(n) \end{bmatrix}$$

$(P^n)_{i,j} = P_{i,j}(n)$ = probability that the system is in state j after n transitions if it starts out in state i.

$$P^n = \begin{bmatrix} P_{1,1}(n) & P_{1,2}(n) & P_{1,3}(n) \\ P_{2,1}(n) & P_{2,2}(n) & \boxed{P_{2,3}(n)} \\ P_{3,1}(n) & P_{3,2}(n) & P_{3,3}(n) \end{bmatrix}$$

$P_{2,3}(n)$: probability that after n transitions, the system will be in state 3 if it starts in state 2.

$(P^n)_{i,j} = P_{i,j}(n)$ = probability that the system is in state j after n transitions if it starts out in state i.