Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|&|&|\\B_1&B_2&B_3\\|&|&|\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|\\B_1\\|\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix}\longrightarrow\begin{bmatrix}1&0&0&2\\0&1&0&0\\0&0&1&-1\end{bmatrix}$$

9

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|&|&|\\B_1&B_2&B_3\\|&|&|\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|\\B_1\\|\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix}\longrightarrow\begin{bmatrix}1&0&0&2\\0&1&0&0\\0&0&1&-1\end{bmatrix}$$

$x_1 = 2$
$x_2 = 0$
$x_3 = -1$

10

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|&|&|\\B_1&B_2&B_3\\|&|&|\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|\\B_1\\|\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix}\longrightarrow\begin{bmatrix}1&0&0&2\\0&1&0&0\\0&0&1&-1\end{bmatrix}$$

$x_1 = 2$
$x_2 = 0$
$x_3 = -1$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

The first column of the inverse has been found. Notice that this column shows up right in the reduced matrix.

11

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|&|&|\\B_1&B_2&B_3\\|&|&|\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|\\B_1\\|\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix}\longrightarrow\begin{bmatrix}1&0&0&2\\0&1&0&0\\0&0&1&-1\end{bmatrix}$$

$x_1 = 2$
$x_2 = 0$
$x_3 = -1$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}2\\0\\-1\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

The first column of the inverse has been found. Notice that this column shows up right in the reduced matrix. The same process can be run for the second and third columns.

12

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|&|&|\\B_1&B_2&B_3\\|&|&|\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|\\B_1\\|\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix}\to \text{RR}$$

What about B_2?

$$\begin{bmatrix}1&2&1&0\\0&1&0&1\\1&1&2&0\end{bmatrix}\to \text{RR \& select last column}$$

13

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|&|&|\\B_1&B_2&B_3\\|&|&|\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|\\B_1\\|\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix}\to \text{RR}$$

What about B_2?

$$\begin{bmatrix}1&2&1&0\\0&1&0&1\\1&1&2&0\end{bmatrix}\to \text{RR \& select last column}$$

What about B_3?

$$\begin{bmatrix}1&2&1&0\\0&1&0&0\\1&1&2&1\end{bmatrix}\to \text{RR \& select last column}$$

14

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|&|&|\\B_1&B_2&B_3\\|&|&|\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}|\\B_1\\|\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix}\to \text{RR}$$

What about B_2?

$$\begin{bmatrix}1&2&1&0\\0&1&0&1\\1&1&2&0\end{bmatrix}\to \text{RR \& select last column}$$

What about B_3?

$$\begin{bmatrix}1&2&1&0\\0&1&0&0\\1&1&2&1\end{bmatrix}\to \text{RR \& select last column}$$

DO THEM ALL AT ONCE:

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix}\to \text{RR}$$

15

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

16

INVERSES AND THE IDENTITY MATRIX

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I \begin{bmatrix} * & * & \& \\ * & 5 & 2 \\ 2 & \pi & 6 \\ \pi & 1 & 1 \end{bmatrix} = \begin{bmatrix} * & * & \& \\ * & 5 & 2 \\ 2 & \pi & 6 \\ \pi & 1 & 1 \end{bmatrix}$$

Check sizes. Multiplication by I leaves things unchanged.

ERASE 1

INVERSES AND THE IDENTITY MATRIX

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I \begin{bmatrix} * & * & \& \\ * & 5 & 2 \\ 2 & \pi & 6 \\ \pi & 1 & 1 \end{bmatrix} = \begin{bmatrix} * & * & \& \\ * & 5 & 2 \\ 2 & \pi & 6 \\ \pi & 1 & 1 \end{bmatrix}$$

Check sizes. Multiplication by I leaves things unchanged.

Definition: Let A be an n x n matrix. Then A is said to have an inverse if there exists a matrix B such that AB = I.

Notation: $A^{-1} = B$.

Fact: If A^{-1} exists, then $AA^{-1} = I = A^{-1}A$.

Problem: How do you find the inverse???

ERASE 2

Find the inverse of $\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}$.

ERASE 3

Find the inverse of $\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}$.

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | & | & | \\ B_1 & B_2 & B_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}.$$

ERASE 4

Find the inverse of $\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}$.

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | & | & | \\ B_1 & B_2 & B_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}.$$

RECALL:

$$\begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ Av_1 & Av_2 & \cdots & Av_n \\ | & | & & | \end{bmatrix}$$

ERASE 5

Find the inverse of $\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}$.

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | & | & | \\ B_1 & B_2 & B_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}.$$

RECALL:

$$\begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix} = \begin{bmatrix} | & | & & | \\ Av_1 & Av_2 & \cdots & Av_n \\ | & | & & | \end{bmatrix}$$

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | \\ B_1 \\ | \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \text{ OR}$$

ERASE 6

Find the inverse of $\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}$.

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | & | & | \\ B_1 & B_2 & B_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}.$$

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | \\ B_1 \\ | \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

ERASE 7

Find the inverse of $\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}$.

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | & | & | \\ B_1 & B_2 & B_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix}.$$

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} | \\ B_1 \\ | \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \text{ OR}$$

$$\begin{bmatrix} 1&2&1 \\ 0&1&0 \\ 1&1&2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1&2&1&1 \\ 0&1&0&0 \\ 1&1&2&0 \end{bmatrix} \to \text{RR}$$

ERASE 8

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix}$$

17

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix}$$

STEP II: Now row reduce.

$(-1)R_1 + R_3$:

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\0&-1&1&-1&0&1\end{bmatrix}$$

$R_2 + R_3$:

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

$(-2)R_2 + R_1$:

$$\begin{bmatrix}1&0&1&1&-2&0\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

$(-1)R_3 + R_1$:

$$\begin{bmatrix}1&0&0&2&-3&-1\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

18

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}1&0&0&2&-3&-1\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

STEP II: Now row reduce.

19

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}1&0&0&2&-3&-1\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

STEP II: Now row reduce.

RECALL

$$\begin{bmatrix}1&2&1&1\\0&1&0&0\\1&1&2&0\end{bmatrix} \longrightarrow \begin{bmatrix}1&0&0&2\\0&1&0&0\\0&0&1&-1\end{bmatrix}$$

So $\begin{bmatrix}2\\0\\-1\end{bmatrix}$ is first column of the inverse

20

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}1&0&0&2&-3&-1\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

STEP II: Now row reduce.

The same reasoning applies to the second and third columns.

21

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}1&0&0&2&-3&-1\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

STEP II: Now row reduce. HERE IS THE INVERSE

22

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}1&0&0&2&-3&-1\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

STEP II: Now row reduce.

CHECK:

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}2&-3&-1\\0&1&0\\-1&1&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

THE INVERSE HAS BEEN FOUND

23

Find the inverse of $\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}$.

STEP I: Augment the matrix with the identity matrix (we just did this directly above).

$$\begin{bmatrix}1&2&1&1&0&0\\0&1&0&0&1&0\\1&1&2&0&0&1\end{bmatrix} \longrightarrow \begin{bmatrix}1&0&0&2&-3&-1\\0&1&0&0&1&0\\0&0&1&-1&1&1\end{bmatrix}$$

STEP II: Now row reduce.

CHECK:

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}2&-3&-1\\0&1&0\\-1&1&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

THE INVERSE HAS BEEN FOUND

Also:

$$\begin{bmatrix}2&-3&-1\\0&1&0\\-1&1&1\end{bmatrix}\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

24

Find the inverse of $\begin{bmatrix}0&1&0\\1&2&1\\2&3&2\end{bmatrix}$.

Find the inverse of $\begin{bmatrix}0&1&0\\1&2&1\\2&3&2\end{bmatrix}$.

STEP I:

$$\begin{bmatrix}0&1&0&1&0&0\\1&2&1&0&1&0\\2&3&2&0&0&1\end{bmatrix}$$

STEP II: Row reduce.

$R_1 \leftrightarrow R_2$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\2&3&2&0&0&1\end{bmatrix}$$

$(-2)R_1 + R_3$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\0&-1&0&0&-2&1\end{bmatrix}$$

$R_2 + R_3$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\0&0&0&1&-2&1\end{bmatrix}$$

Find the inverse of $\begin{bmatrix}0&1&0\\1&2&1\\2&3&2\end{bmatrix}$.

STEP I:

$$\begin{bmatrix}0&1&0&1&0&0\\1&2&1&0&1&0\\2&3&2&0&0&1\end{bmatrix}$$

STEP II: Row reduce.

$R_1 \leftrightarrow R_2$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\2&3&2&0&0&1\end{bmatrix}$$

$(-2)R_1 + R_3$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\0&-1&0&0&-2&1\end{bmatrix}$$

$R_2 + R_3$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\0&0&0&1&-2&1\end{bmatrix}$$

$0x_1 + 0x_2 + 0x_3 = 1$

INCONSISTENT

There is no solution. This matrix has no inverse.

Find the inverse of $\begin{bmatrix}0&1&0\\1&2&1\\2&3&2\end{bmatrix}$.

STEP I:

$$\begin{bmatrix}0&1&0&1&0&0\\1&2&1&0&1&0\\2&3&2&0&0&1\end{bmatrix}$$

STEP II: Row reduce.

$R_1 \leftrightarrow R_2$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\2&3&2&0&0&1\end{bmatrix}$$

$(-2)R_1 + R_3$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\0&-1&0&0&-2&1\end{bmatrix}$$

$R_2 + R_3$:

$$\begin{bmatrix}1&2&1&0&1&0\\0&1&0&1&0&0\\0&0&0&1&-2&1\end{bmatrix}$$

$$\begin{bmatrix}0&1&0\\1&2&1\\2&3&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}.$$

This has no solution. But the solution is the first column of the inverse.

Consider the following system of equations:

$$\begin{aligned}1x_1 \; +2x_2 \; +1x_3 &= 5\\ 0x_1 \; +1x_2 \; +0x_3 &= 10\\ 1x_1 \; +1x_2 \; +2x_3 &= 7\end{aligned}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}5\\10\\7\end{bmatrix}$$

A x = b

$Ax = b$.

One way to solve this MATRIX EQUATION is to solve the corresponding (i.e. equivalent) system of equations given above. To do this, form the augmented matrix:

Consider the following system of equations:

$$\begin{aligned}1x_1 \; +2x_2 \; +1x_3 &= 5\\ 0x_1 \; +1x_2 \; +0x_3 &= 10\\ 1x_1 \; +1x_2 \; +2x_3 &= 7\end{aligned}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}5\\10\\7\end{bmatrix}$$

A x = b

$Ax = b$.

One way to solve this MATRIX EQUATION is to solve the corresponding (i.e. equivalent) system of equations given above. To do this, form the augmented matrix:

$$\begin{bmatrix}1&2&1&5\\0&1&0&10\\1&1&2&7\end{bmatrix}$$

and row reduce.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}5\\10\\7\end{bmatrix}$$

A x = b

$Ax=b$.

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}5\\10\\7\end{bmatrix}$$

A x = b

$Ax=b$.

Multiply both sides by A^{-1} provided it exists

In this case, it does.

$A^{-1}(Ax) = A^{-1}b \Rightarrow$

$(A^{-1}A)x = A^{-1}b \Rightarrow$

$(I)x = A^{-1}b \Rightarrow$

$x = A^{-1}b$. DONE

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}5\\10\\7\end{bmatrix}$$

A x = b

Ax=b.

Multiply both sides by A^{-1} provided it exists
In this case, it does.

$A^{-1}(Ax) = A^{-1}b \Rightarrow$

$(A^{-1}A)x = A^{-1}b \Rightarrow$

$(I)x = A^{-1}b \Rightarrow$

$x = A^{-1}b$. DONE

WE KNOW WHAT A^{-1} IS:

$$A^{-1}=\begin{bmatrix}2&-3&-1\\0&1&0\\-1&1&1\end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix}2&-3&-1\\0&1&0\\-1&1&1\end{bmatrix}\begin{bmatrix}5\\10\\7\end{bmatrix} = \begin{bmatrix}-27\\10\\12\end{bmatrix}$$

$$\begin{bmatrix}1&2&1\\0&1&0\\1&1&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}5\\10\\7\end{bmatrix}$$

A x = b

Ax=b.

Multiply both sides by A^{-1} provided it exists
In this case, it does.

$A^{-1}(Ax) = A^{-1}b \Rightarrow$

$(A^{-1}A)x = A^{-1}b \Rightarrow$

$(I)x = A^{-1}b \Rightarrow$

$x = A^{-1}b$. DONE

WE KNOW WHAT A^{-1} IS:

$$A^{-1}=\begin{bmatrix}2&-3&-1\\0&1&0\\-1&1&1\end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix}2&-3&-1\\0&1&0\\-1&1&1\end{bmatrix}\begin{bmatrix}5\\10\\7\end{bmatrix} = \begin{bmatrix}-27\\10\\12\end{bmatrix}$$

COMMENT:

BY SOLVING 3 EQS., WE WERE ABLE TO SOLVE ALL OTHERS.

Comments about Inverses:

1) Not all matrices are invertible. A matrix is invertible iff it row reduces to the identity matrix. Only square matrices can do this and NOT all square matrices do this.

2) $(A^{-1})^{-1} = A$

3) ABCDE is invertible iff each of the matrices in the product is invertible.

4) $(AB)^{-1} = B^{-1}A^{-1}$

EXAMPLE: Solve

$$2x_1 + x_2 = 1$$
$$3x_1 - x_2 = 3 .$$

EXAMPLE: Solve

$$2x_1 + x_2 = 1$$
$$3x_1 - x_2 = 3 .$$

Rewritten as a matrix equation this becomes:

$$\begin{bmatrix}2&1\\3&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$$

$Ax = b$
$x = A^{-1}b$

EXAMPLE: Solve

$$2x_1 + x_2 = 1$$
$$3x_1 - x_2 = 3 .$$

Rewritten as a matrix equation this becomes:

$$\begin{bmatrix}2&1\\3&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$$

$Ax = b$
$x = A^{-1}b$

Find A^{-1}:

$$\begin{bmatrix}2&1&1&0\\3&-1&0&1\end{bmatrix}$$ Row reduce & find A^{-1}.

EXAMPLE: Solve

$$2x_1 + x_2 = 1$$
$$3x_1 - x_2 = 3 .$$

Rewritten as a matrix equation this becomes:

$$\begin{bmatrix}2&1\\3&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$$

$Ax = b$
$x = A^{-1}b$

Find A^{-1}:

$$\begin{bmatrix}2&1&1&0\\3&-1&0&1\end{bmatrix}$$ Row reduce & find A^{-1}.

$$A^{-1}=\begin{bmatrix}\frac{1}{5}&\frac{1}{5}\\\frac{3}{5}&-\frac{2}{5}\end{bmatrix}.$$

EXAMPLE: Solve

$$2x_1 + x_2 = 1$$
$$3x_1 - x_2 = 3 .$$

Rewritten as a matrix equation this becomes:

$$\begin{bmatrix}2&1\\3&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\3\end{bmatrix}$$

$Ax = b$
$x = A^{-1}b$

Find A^{-1}:

$$\begin{bmatrix}2&1&1&0\\3&-1&0&1\end{bmatrix}$$ Row reduce & find A^{-1}.

$$A^{-1}=\begin{bmatrix}\frac{1}{5}&\frac{1}{5}\\\frac{3}{5}&-\frac{2}{5}\end{bmatrix}.$$

And

$$x = A^{-1}b = \begin{bmatrix}\frac{1}{5}&\frac{1}{5}\\\frac{3}{5}&-\frac{2}{5}\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}\frac{4}{5}\\-\frac{3}{5}\end{bmatrix}$$

So

$x_1 = \frac{4}{5}$

$x_2 = -\frac{3}{5}$

EXAMPLE: Solve

$$2x_1 + x_2 = 5$$
$$3x_1 - x_2 = 10.$$

41

EXAMPLE: Solve

$$2x_1 + x_2 = 5$$
$$3x_1 - x_2 = 10.$$

$$x = A^{-1}b = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{3}{5} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$x_1 = 3$$
$$x_2 = -1$$

42

Problem: Find C such that

$$\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} C = \begin{bmatrix} 10 & 5 \\ 10 & 15 \end{bmatrix}.$$

43

Problem: Find C such that

$$\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} C = \begin{bmatrix} 10 & 5 \\ 10 & 15 \end{bmatrix}.$$

Answer:

$$C = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{3}{5} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} 10 & 5 \\ 10 & 15 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 2 & -3 \end{bmatrix}.$$

44

45

46

47

48