DEFINITION:

A matrix is an m x n array of numbers.

Example: 3 x 4 (#Rows x #Columns)

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ \pi & 1 & \frac{1}{2} & -8 \\ 0 & 0 & 7 & 0 \end{bmatrix}$$

The (2,3) entry is $\frac{1}{2}$.

The (i,j) entry is the entry in the i th row and j th column.

NOTATION: If A is a matrix, the (a_{ij}) will be used sometimes to denote this matrix.

NOTATION: If A is a matrix, then (a will be used sometimes to denote this matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & a_{m3} & . & . & a_{mn} \end{bmatrix}$$

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EXAMPLES:

Column vectors:

$$\begin{bmatrix} 6 \\ -2 \\ 1/45 \\ 10 \end{bmatrix} \quad 4 \times 1$$

Row vectors:

$$\begin{bmatrix} 5 & 7 & -3 & 1.02 & 0 \end{bmatrix} \quad 1 \times 5$$

General matrices:

$$\begin{bmatrix} 2 & 7 & 1 & 9 \\ -8 & 95 & 1/3 & -50 \end{bmatrix} \quad 2 \times 4$$

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ADDITION:

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 6 \\ 1 & 1 & 1 \\ 2 & -2 & 7 \end{bmatrix} + \begin{bmatrix} -2 & 3 & 6 \\ 3 & 7 & 1 \\ 0 & 0 & 0 \\ 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 10 \\ 3 & 11 & 12 \\ 1 & 1 & 1 \\ 5 & 1 & 8 \end{bmatrix}$$

All matrices the same size.

SCALAR MULTIPLICATION:

$$3 \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 6 \\ 1 & 1 & 1 \\ 2 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 15 \\ 0 & 12 & 18 \\ 3 & 3 & 3 \\ 6 & -6 & 21 \end{bmatrix}$$

All matrices the same size.

PROPERTIES:

$A + B = B + A$
$cA + dA = (c+d)A$
$cA + cB = c(A+B)$
$(cd)A = c(dA)$

All matrices the same size in each equation.

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RECALL EARLIER EXAMPLE:

$1x_1 - 1x_2 + 0x_3 - 2x_4 = 3$
$2x_1 - 2x_2 + 2x_3 - 2x_4 = 4$
$1x_3 + 1x_4 = -1$

ASSOCIATED AUGMENTED MATRIX:

$$\begin{bmatrix} 1 & -1 & 0 & -2 & 3 \\ 2 & -2 & 2 & -2 & 4 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$-2R_1 + R_2$:

$$\begin{bmatrix} 1 & -1 & 0 & -2 & 3 \\ 0 & 0 & 2 & 2 & -2 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

(1/2) R_2:

$$\begin{bmatrix} 1 & -1 & 0 & -2 & 3 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$-1R_2 + R_3$:

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 & \\ \end{matrix}$$
$$\begin{bmatrix} 1 & -1 & 0 & -2 & 3 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The variables x_2 and x_4 variables are free. Solve for the other variables in terms of these.

$x_3 + x_4 = -1 \Rightarrow x_3 = -1 - x_4$

$x_1 - x_2 - 2x_4 = 3 \Rightarrow x_1 = 3 + x_2 + 2x_4$

$$\begin{matrix} x_1 \to \\ x_2 \to \\ x_3 \to \\ x_4 \to \end{matrix} \begin{bmatrix} 3 + x_2 + 2x_4 \\ x_2 \\ -1 - x_4 \\ x_4 \end{bmatrix}$$

Solution written as a column matrix.

where x_2 and x_4 are free to be any real numbers.

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$$\begin{bmatrix} 3 + x_2 + 2x_4 \\ x_2 \\ -1 - x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2x_4 \\ 0 \\ -1x_4 \\ 1x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Definition of matrix addition — Definition of scalar multiplication

The solution consists of the point $\begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ in 4 dimensional space, and all other points that can be found by travelling out in the directions

$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$. This describes a plane in 4 dimensional space.

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$$\begin{bmatrix} 3 + x_2 + 2x_4 \\ x_2 \\ -1 - x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2x_4 \\ 0 \\ -1x_4 \\ 1x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Definition of matrix addition — Definition of scalar multiplication

One solution is the point $\begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix}$. Here is another one: Set $x_2 = 10$ and $x_4 = 20$. Then

$$\begin{bmatrix} 3 \\ 0 \\ -1 \\ 0 \end{bmatrix} + 10 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 20 \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 53 \\ 10 \\ -21 \\ 20 \end{bmatrix}$$

is another solution.

$x_1 = 53$, $x_2 = 10$, $x_3 = -21$, $x_4 = 20$.

Every choice of x_2 and x_4 will yield a solution.

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A matrix times a column vector:

Example:

Call ↓ this matrix E:

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 7 \\ 2 & 1 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 8 \\ 2 \end{bmatrix}$$

4 x 3   3 x 1   4 x 1

$1 \cdot 1 + 2 \cdot 0 + 4 \cdot 1 = 5$

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A matrix times a column vector:

Example:

Call ↓ this matrix E:

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 7 \\ 2 & 1 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 8 \\ 2 \end{bmatrix}$$

4 x 3   3 x 1   4 x 1

Notice:

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 7 \\ 2 & 1 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

E c^1 $E_{\cdot 1}$

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A matrix times a column vector:

Example:

Call ↓ this matrix E:

$$\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}5\\10\\8\\2\end{bmatrix}$$

4 x 3 3 x 1 4 x 1

Notice:

$$\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}\underset{c^1}{\begin{bmatrix}1\\0\\0\end{bmatrix}}=\underset{E_{\bullet 1}}{\begin{bmatrix}1\\3\\2\\1\end{bmatrix}}$$

$$\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}\underset{c^2}{\begin{bmatrix}0\\1\\0\end{bmatrix}}=\underset{E_{\bullet 2}}{\begin{bmatrix}2\\1\\1\\1\end{bmatrix}}$$

$$\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}\underset{c^3}{\begin{bmatrix}0\\0\\1\end{bmatrix}}=\underset{E_{\bullet 3}}{\begin{bmatrix}4\\7\\6\\1\end{bmatrix}}$$

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A row vector times a matrix:

Example:

$$\begin{bmatrix}1&0&2&3\end{bmatrix}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\begin{bmatrix}8&7&19\end{bmatrix}$$

1 x 4 4x3 1 x 3

Notice:

$$\underset{r_1}{\begin{bmatrix}1&0&0&0\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{1\bullet}}{\begin{bmatrix}1&2&4\end{bmatrix}}$$

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A row vector times a matrix:

Example:

$$\begin{bmatrix}1&0&2&3\end{bmatrix}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\begin{bmatrix}8&7&19\end{bmatrix}$$

1 x 4 4x3 1 x 3

Notice:

$$\underset{r_1}{\begin{bmatrix}1&0&0&0\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{1\bullet}}{\begin{bmatrix}1&2&4\end{bmatrix}}$$

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$$\underset{r_2}{\begin{bmatrix}0&1&0&0\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{2\bullet}}{\begin{bmatrix}3&1&7\end{bmatrix}}$$

$$\underset{r_3}{\begin{bmatrix}0&0&1&0\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{3\bullet}}{\begin{bmatrix}2&1&6\end{bmatrix}}$$

$$\underset{r_4}{\begin{bmatrix}0&0&0&1\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{4\bullet}}{\begin{bmatrix}1&1&1\end{bmatrix}}$$

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$$\underset{r_2}{\begin{bmatrix}0&1&0&0\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{2\bullet}}{\begin{bmatrix}3&1&7\end{bmatrix}}$$

$$\underset{r_3}{\begin{bmatrix}0&0&1&0\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{3\bullet}}{\begin{bmatrix}2&1&6\end{bmatrix}}$$

$$\underset{r_4}{\begin{bmatrix}0&0&0&1\end{bmatrix}}\underset{E}{\begin{bmatrix}1&2&4\\3&1&7\\2&1&6\\1&1&1\end{bmatrix}}=\underset{E_{4\bullet}}{\begin{bmatrix}1&1&1\end{bmatrix}}$$

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MATRIX MULTIPLICATION:

$$\underset{2\times 3}{\begin{bmatrix}1&2&1\\3&1&2\end{bmatrix}}\underset{3\times 2}{\begin{bmatrix}3&7\\4&8\\1&1\end{bmatrix}}=\underset{2\times 2}{\begin{bmatrix}\square&\square\\\square&\square\end{bmatrix}}$$

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MATRIX MULTIPLICATION:

$$\underset{2\times 3}{\begin{bmatrix}1&2&1\\3&1&2\end{bmatrix}}\underset{3\times 2}{\begin{bmatrix}3&7\\4&8\\1&1\end{bmatrix}}=\underset{2\times 2}{\begin{bmatrix}\square&????\\\square&\square\end{bmatrix}}$$

← (1, 2) ENTRY
ROW 1
COLUMN 2

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MATRIX MULTIPLICATION:

$$\underset{\text{ROW 1}}{\begin{bmatrix}1&2&1\\3&1&2\end{bmatrix}}\underset{\text{COLUMN 2}}{\begin{bmatrix}3&7\\4&8\\1&1\end{bmatrix}}=\underset{2\times 2}{\begin{bmatrix}\square&????\\\square&\square\end{bmatrix}}$$

← (1,2) ENTRY
ROW 1
COLUMN 2

$1\cdot 7 + 2\cdot 8 + 1\cdot 1 = 24$

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MATRIX MULTIPLICATION:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} & 24 \\ & ???? \end{bmatrix}$$

ROW 2 COLUMN 2

2 x 2

(2,2) ENTRY
ROW 2
COLUMN 2

$3 \cdot 7 + 1 \cdot 8 + 2 \cdot 1 = 31$

MATRIX MULTIPLICATION:

(1,1) ENTRY
ROW 1
COLUMN 1

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} ???? & 24 \\ & 31 \end{bmatrix}$$

ROW 1 COLUMN 1

2 x 2

$1 \cdot 3 + 2 \cdot 4 + 1 \cdot 1 = 12$

MATRIX MULTIPLICATION:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ ???? & 31 \end{bmatrix}$$

ROW 2 COLUMN 1

(2,1) ENTRY
ROW 2
COLUMN 1

$3 \cdot 3 + 1 \cdot 4 + 2 \cdot 1 = 15$

MATRIX MULTIPLICATION:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

MATRIX MULTIPLICATION:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3 3 x 2 2 x 2

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix}$$

MATRIX MULTIPLICATION:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3 3 x 2 2 x 2

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 24 \\ 31 \end{bmatrix}$$

MATRIX MULTIPLICATION:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3 3 x 2 2 x 2

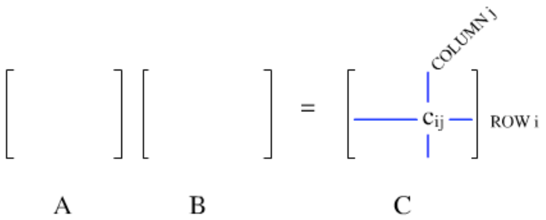
$$\begin{bmatrix} & A & \end{bmatrix} \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} = \begin{bmatrix} | & | \\ Av_1 & Av_2 \\ | & | \end{bmatrix}$$

MATRIX MULTIPLICATION:

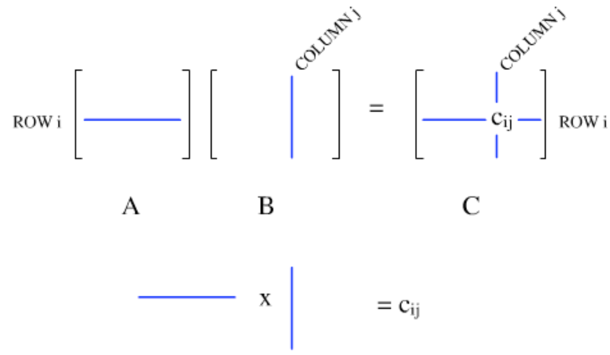
$$\begin{bmatrix} \end{bmatrix} \begin{bmatrix} \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$

A B C

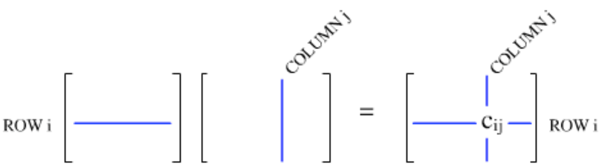
MATRIX MULTIPLICATION:



MATRIX MULTIPLICATION:



MATRIX MULTIPLICATION:



$$\begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} = \begin{bmatrix} Av_1 & Av_2 & \cdots & Av_n \end{bmatrix}$$

EARLIER EXAMPLE:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3   3 x 2   2 x 2

NEXT TRY:

$$\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

3 x 2   2 x 3

EARLIER EXAMPLE:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3   3 x 2   2 x 2

NEXT TRY:

$$\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

3 x 2   2 x 3   3 x 3

EARLIER EXAMPLE:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3   3 x 2   2 x 2

NEXT TRY:

$$\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & ?? \\ \square & \square & \square \end{bmatrix}$$

3 x 2   2 x 3   3 x 3

EARLIER EXAMPLE:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3   3 x 2   2 x 2

NEXT TRY:

$$\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & 4\cdot1+8\cdot2 \\ \square & \square & \square \end{bmatrix}$$

3 x 2   2 x 3   3 x 3

EARLIER EXAMPLE:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 24 \\ 15 & 31 \end{bmatrix}$$

2 x 3   3 x 2   2 x 2

NEXT TRY:

$$\begin{bmatrix} 3 & 7 \\ 4 & 8 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square \\ \square & \square & 20 \\ \square & \square & \square \end{bmatrix}$$

3 x 2   2 x 3   3 x 3

EARLIER EXAMPLE:

$$\begin{bmatrix}1 & 2 & 1\\3 & 1 & 2\end{bmatrix}\begin{bmatrix}3 & 7\\4 & 8\\1 & 1\end{bmatrix} = \begin{bmatrix}12 & 24\\15 & 31\end{bmatrix}$$

2 x 3 3 x 2 2 x 2

NEXT TRY:

$$\begin{bmatrix}3 & 7\\4 & 8\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\3 & 1 & 2\end{bmatrix} = \begin{bmatrix}24 & 13 & 17\\28 & 16 & 20\\4 & 3 & 3\end{bmatrix}$$

3 x 2 2 x 3 3 x 3

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EARLIER EXAMPLE:

$$\begin{bmatrix}1 & 2 & 1\\3 & 1 & 2\end{bmatrix}\begin{bmatrix}3 & 7\\4 & 8\\1 & 1\end{bmatrix} = \begin{bmatrix}12 & 24\\15 & 31\end{bmatrix}$$

2 x 3 3 x 2 2 x 2

MATRIX MULTIPLICATION IS NOT COMMUTATIVE!

NEXT TRY:

$$\begin{bmatrix}3 & 7\\4 & 8\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\3 & 1 & 2\end{bmatrix} = \begin{bmatrix}24 & 13 & 17\\28 & 16 & 20\\4 & 3 & 3\end{bmatrix}$$

3 x 2 2 x 3 3 x 3

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EXAMPLE:

$$\begin{bmatrix}A\end{bmatrix}\begin{bmatrix}B\end{bmatrix} = \begin{bmatrix}\ \end{bmatrix}$$

2 x 3 3 x 4 2 x 4

NEXT TRY:

$$\begin{bmatrix}B\end{bmatrix}\begin{bmatrix}A\end{bmatrix} =$$ DOESN'T MAKE SENSE!!!

3 x 4 2 x 3

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EXAMPLE:

$$\begin{bmatrix}A\end{bmatrix}\begin{bmatrix}B\end{bmatrix} = \begin{bmatrix}\ \end{bmatrix}$$

2 x 3 3 x 4 2 x 4

NEXT TRY:

$$\begin{bmatrix}B\end{bmatrix}\begin{bmatrix}A\end{bmatrix} =$$ NOT ONLY DOES $AB \neq BA$ BUT BA DOESN'T EVEN EXIST.

3 x 4 2 x 3

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EVEN IN SIMPLE SITUATIONS COMMUTATIVITY DOES NOT HOLD:

$$\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0 & 4 & 2\\1 & -2 & 0\\2 & 12 & 8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\3\\6\end{bmatrix}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0 & 4 & 2\\1 & -2 & 0\\2 & 12 & 8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\3\\6\end{bmatrix}$$

CALCULATE THE PRODUCT ON THE LEFT HAND SIDE

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0 & 4 & 2\\1 & -2 & 0\\2 & 12 & 8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$\begin{bmatrix}0x_1 + 4x_2 + 2x_3\\1x_1 - 2x_2 + 0x_3\\2x_1 + 12x_2 + 8x_3\end{bmatrix} = \begin{bmatrix}0\\3\\6\end{bmatrix}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$\begin{bmatrix}0x_1+4x_2+2x_3\\1x_1-2x_2+0x_3\\2x_1+12x_2+8x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

TWO MATRICES ARE EQUAL ONLY IF THEIR (RESPECTIVE) ENTRIES ARE EQUAL

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$\begin{bmatrix}0x_1+4x_2+2x_3\\1x_1-2x_2+0x_3\\2x_1+12x_2+8x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

SO:

$$\begin{aligned}0x_1+4x_2+2x_3&=0\\1x_1-2x_2+0x_3&=3\\2x_1+12x_2+8x_3&=6\end{aligned}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$\begin{bmatrix}0x_1+4x_2+2x_3\\1x_1-2x_2+0x_3\\2x_1+12x_2+8x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

CORRESPONDENCE BETWEEN MATRIX EQUATION AND A SYSTEM OF LINEAR EQUATIONS

SO:

$$\begin{aligned}0x_1+4x_2+2x_3&=0\\1x_1-2x_2+0x_3&=3\\2x_1+12x_2+8x_3&=6\end{aligned}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$\begin{bmatrix}0x_1+4x_2+2x_3\\1x_1-2x_2+0x_3\\2x_1+12x_2+8x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

SO:

$$\begin{aligned}0x_1+4x_2+2x_3&=0\\1x_1-2x_2+0x_3&=3\\2x_1+12x_2+8x_3&=6\end{aligned}\rightarrow\begin{bmatrix}0&4&2&0\\1&-2&0&3\\2&12&8&6\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&3\\0&1&.5&0\\0&0&0&0\end{bmatrix}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$x_1 = 3 - x_3$$
$$x_2 = -(1/2)x_3$$

SO:

$$\begin{aligned}0x_1+4x_2+2x_3&=0\\1x_1-2x_2+0x_3&=3\\2x_1+12x_2+8x_3&=6\end{aligned}\rightarrow\begin{bmatrix}0&4&2&0\\1&-2&0&3\\2&12&8&6\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&3\\0&1&.5&0\\0&0&0&0\end{bmatrix}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$\begin{aligned}x_1 &= 3 - x_3\\x_2 &= -(1/2)x_3\end{aligned}\rightarrow\begin{bmatrix}3-x_3\\-(1/2)x_3\\x_3\end{bmatrix}$$

FEED IN ANY VALUE FOR x_3 AND YOU WILL GET A SOLUTION TO MATRIX EQUATION

SO:

$$\begin{aligned}0x_1+4x_2+2x_3&=0\\1x_1-2x_2+0x_3&=3\\2x_1+12x_2+8x_3&=6\end{aligned}\rightarrow\begin{bmatrix}0&4&2&0\\1&-2&0&3\\2&12&8&6\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&3\\0&1&.5&0\\0&0&0&0\end{bmatrix}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}\qquad\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}-7\\-5\\10\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

$$\begin{aligned}x_1 &= 3 - x_3\\x_2 &= -(1/2)x_3\end{aligned}\rightarrow\begin{bmatrix}3-10\\-(1/2)10\\10\end{bmatrix}$$

EXAMPLE: $x_3 = 10$

SO:

$$\begin{aligned}0x_1+4x_2+2x_3&=0\\1x_1-2x_2+0x_3&=3\\2x_1+12x_2+8x_3&=6\end{aligned}\rightarrow\begin{bmatrix}0&4&2&0\\1&-2&0&3\\2&12&8&6\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&3\\0&1&.5&0\\0&0&0&0\end{bmatrix}$$

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EXAMPLE: Consider the following matrix equation:

$$\begin{bmatrix}0&4&2\\1&-2&0\\2&12&8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\3\\6\end{bmatrix}$$

SKIP TRANSFERING OVER TO A SYSTEM OF EQUATIONS

$$\rightarrow\begin{bmatrix}0&4&2&0\\1&-2&0&3\\2&12&8&6\end{bmatrix}\rightarrow\begin{bmatrix}1&0&1&3\\0&1&.5&0\\0&0&0&0\end{bmatrix}$$

$$\rightarrow\begin{aligned}x_1 &= 3 - x_3\\x_2 &= -(1/2)x_3\end{aligned}\rightarrow\begin{bmatrix}3-x_3\\-(1/2)x_3\\x_3\end{bmatrix}$$

x_3 FREE

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GENERAL PATTERN:

$$\begin{bmatrix} 0 & 4 & 2 \\ 1 & -2 & 0 \\ 2 & 12 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$$

$$A \quad x \quad = \quad b$$

Given A and b, find x.

SKIP TRANSFERING OVER TO A SYSTEM OF EQUATIONS

Solve:

$$\left[\; A \;\middle|\; b \;\right] \longrightarrow \left[\; \text{Reduced Form} \;\right] \longrightarrow \left[\; x \;\right]$$