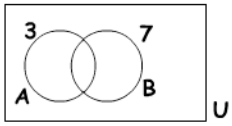
NOTATION: IF **A** IS A SET WITH ONLY A FINITE NUMBER OF ELEMENTS , THEN

n(A) = number of elements in **A**.

EXAMPLE: IF $A=\{1,2,3,a,b,c\}$ THEN $n(A)=6$.

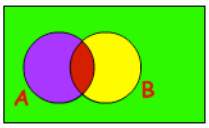
IT WILL BE USEFUL TO LABEL VENN DIAGRAMS WITH THE SIZES OF THE SETS IN THE DIAGRAM:



n(A) = 3
n(B) = 7

1

BUT THIS IS A LITTLE AWKWARD. THINGS WORK OUT BETTER IF YOU KNOW THE SIZES OF ALL THE "PIECES."



WE'LL USE ONE BASIC FACT THROUGHOUT:

$n(E \cup F) = n(E) + n(F)$ ***provided*** $E \cap F = \emptyset$.

NOTE: IF $E \cap F = \emptyset$, THEN E AND F ARE SAID TO BE DISJOINT.

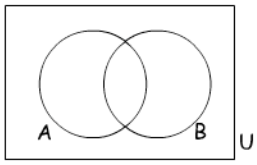
2

Lecture 2

EXAMPLE: Let

n(A∩B) = 7
n(A) = 10
n(B) =18
n(U) = 30.

FILL OUT THE VENN DIAGRAM:



USUALLY THE BEST WAY TO DO THIS IS TO START FROM THE INSIDE AND WORK OUTWARDS.

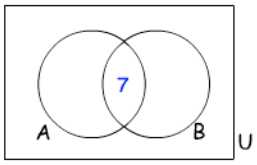
START WITH A∩B.

3

EXAMPLE: Let

n(A∩B) = 7
n(A) = 10
n(B) =18
n(U) = 30.

FILL OUT THE VENN DIAGRAM:



AFTER THIS AIM FOR A - B.

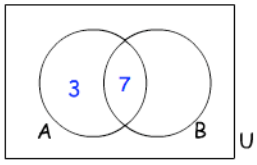
4

EXAMPLE: Let

n(A∩B) = 7
n(A) = 10
n(B) =18
n(U) = 30.

$n(E \cup F) = n(E) + n(F)$ ***provided*** $E \cap F = \emptyset$.

FILL OUT THE VENN DIAGRAM:



n(A - B) + n(A∩B) = n(A)

using the basic fact. So

n(A - B) + 7 = 10 ⇒ n(A - B) =3

NOW TRY FOR n(B - A).

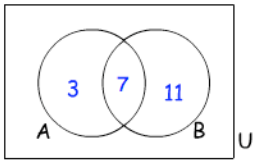
5

EXAMPLE: Let

n(A∩B) = 7
n(A) = 10
n(B) =18
n(U) = 30.

$n(E \cup F) = n(E) + n(F)$ ***provided*** $E \cap F = \emptyset$.

FILL OUT THE VENN DIAGRAM:



n(B - A) + n(A∩B) = n(B)

by the basic fact. So

n(B - A) + 7 = 18 ⇒ n(A - B) =11

NEXT: WHAT IS n(B ∪ A)?

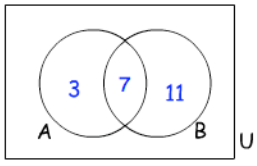
6

EXAMPLE: Let

n(A∩B) = 7
n(A) = 10
n(B) =18
n(U) = 30.

$n(E \cup F) = n(E) + n(F)$ ***provided*** $E \cap F = \emptyset$.

FILL OUT THE VENN DIAGRAM:



$n(A \cup B) = 3 + 7 + 11 = 21$

NOW FILL IN THE REST

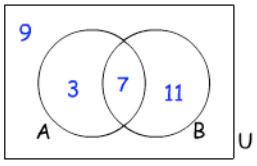
7

EXAMPLE: Let

n(A∩B) = 7
n(A) = 10
n(B) =18
n(U) = 30.

$n(E \cup F) = n(E) + n(F)$ ***provided*** $E \cap F = \emptyset$.

FILL OUT THE VENN DIAGRAM:



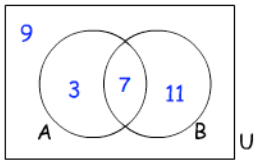
$3 + 7 + 11 + n((A \cup B)^c) = 30$

$\Rightarrow \quad n((A \cup B)^c) = 9$

8

QUESTION: WHAT IS

$n((A \cap B) \cup (A \cup B)^c)$?

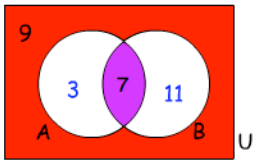


ANSWER: DRAW THE APPROPRIATE VENN DIAGRAM AND COUNT.

9

QUESTION: WHAT IS

$n((A \cap B) \cup (A \cup B)^c)$?



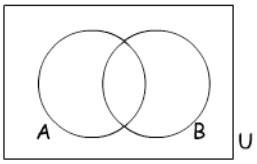
$9 + 7 = \underline{\underline{16}}$

10

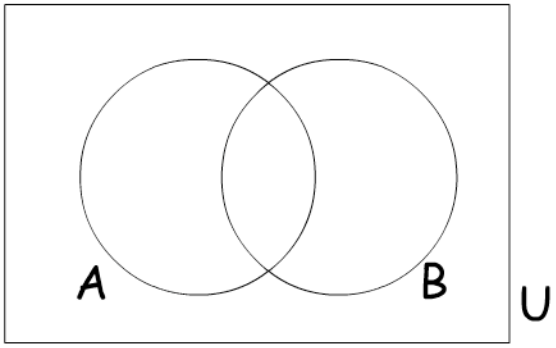
Lecture 2

LET'S DO THE SAME REASONING WE JUST DID, BUT FOR GENERAL SETS WITHOUT USING ACTUAL NUMBERS.

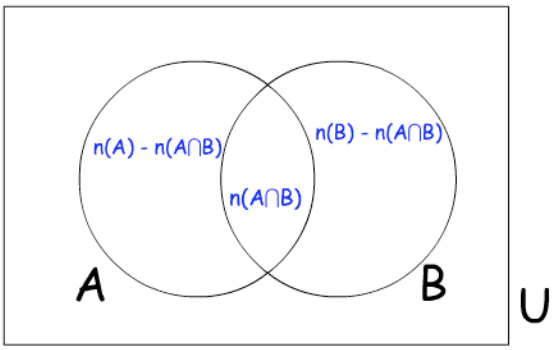
HERE'S A GENERAL VENN DIAGRAM. BLOW IT UP, TO WRITE ON IT BETTER.



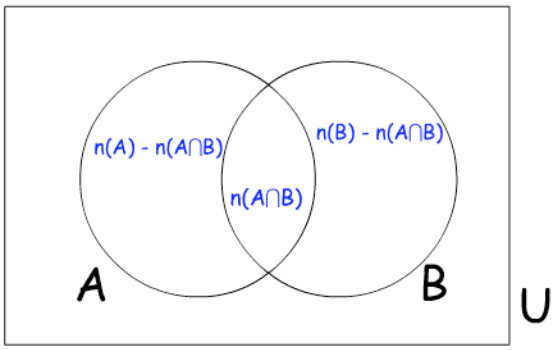
11



13



13



$n(A \cup B) = n(A) - n(A \cap B) + n(A \cap B) + n(B) - n(A \cap B)$

$= n(A) + n(B) - n(A \cap B)$

14

We end up with the formula:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

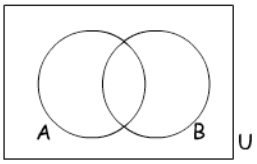
This is consistent with the formula:

$n(A \cup B) = n(A) + n(B)$ for disjoint sets.

Here is another way to derive this formula;

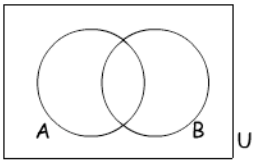
15

Paint the Venn diagram:



Think of each paint molecule as being one element in the set. If you paint a region, then the amount paint used will represent how many elements are in the set (each gallon = so many molecules = so many elements). So measure gallons used.

16

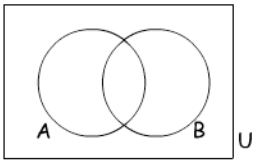


Question: What is n(A ∪ B)? To answer this, paint A ∪ B and measure the amount of paint used.

Next: Paint A ∪ B another way ⇒

17

Paint the Venn diagram:

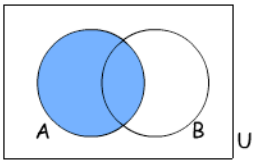


Paint A and then paint B keeping track of how much paint is used:

18

Lecture 2

Paint the Venn diagram:

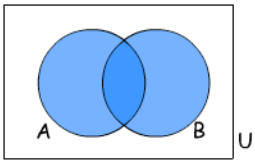


Paint A and then paint B keeping track of how much paint is used:

n(A)

19

Paint the Venn diagram:



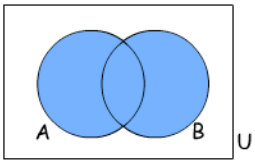
Paint A and then paint B keeping track of how much paint is used:

n(A) + n(B)

NOTICE THAT A∩B HAS TWO LAYERS OF PAINT ON IT. SCRAPE ONE LAYER OFF.

20

Paint the Venn diagram:



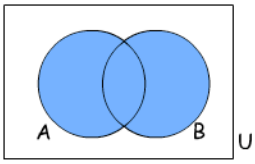
Paint A and then paint B keeping track of how much paint is used:

n(A) + n(B) - n(A∩B)

NOTICE THAT A∩B HAS TWO LAYERS OF PAINT ON IT. SCRAPE ONE LAYER OFF.

21

Paint the Venn diagram:



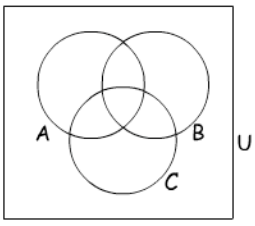
We now have one layer of paint on ALL of A ∪ B. SO...

n(A ∪ B) =

n(A) + n(B) - n(A∩B)

22

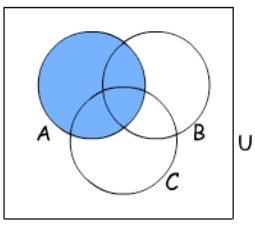
Let's try THREE sets:



What is n(A ∪ B ∪ C)? Answer: Paint A then B then C, and see what happens.

23

Let's try THREE sets:

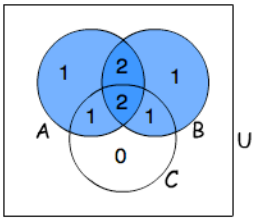


What is n(A ∪ B ∪ C)? Answer: Paint A then B then C, and see what happens.

n(A)

24

Let's try THREE sets:



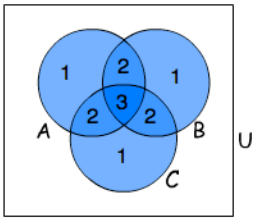
The various regions have been labelled with the number of layers of paint that they have.

What is $n(A \cup B \cup C)$? Answer: Paint A then B then C, and see what happens.

$n(A) + n(B)$

25

Let's try THREE sets:



The various regions have been labelled with the number of layers of paint that they have.

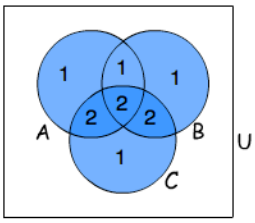
What is $n(A \cup B \cup C)$? Answer: Paint A then B then C, and see what happens.

$n(A) + n(B) + n(C)$

26

Lecture 2

Let's try THREE sets:



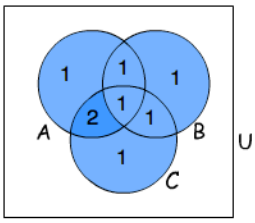
The various regions have been labelled with the number of layers of paint that they have.

What is $n(A \cup B \cup C)$? Now scrape away one layer on $A \cap B$.

$n(A) + n(B) + n(C) - n(A \cap B)$

27

Let's try THREE sets:



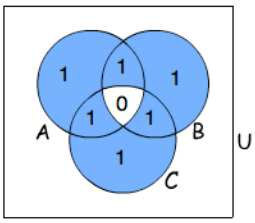
The various regions have been labelled with the number of layers of paint that they have.

What is $n(A \cup B \cup C)$? Now scrape away one layer on $B \cap C$.

$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$

28

Let's try THREE sets:



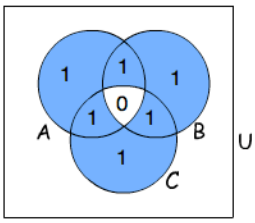
The various regions have been labelled with the number of layers of paint that they have.

What is $n(A \cup B \cup C)$? Now scrape away one layer on $A \cap C$.

$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$

29

Let's try THREE sets:



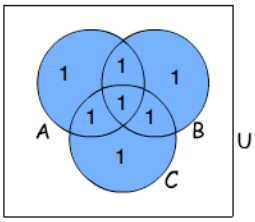
The various regions have been labelled with the number of layers of paint that they have.

What is $n(A \cup B \cup C)$? OH NO!!! There is a bare patch $A \cap B \cap C$. That's OK just repaint it.

$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$

30

Let's try THREE sets:



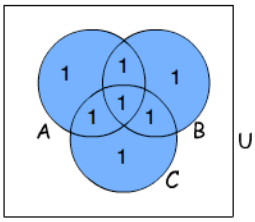
The various regions have been labelled with the number of layers of paint that they have.

What is $n(A \cup B \cup C)$? Now $A \cup B \cup C$ is covered completely with one layer of paint.

$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

31

Let's try THREE sets:



The various regions have been labelled with the number of layers of paint that they have.

CONCLUSION:

$n(A \cup B \cup C) =$

$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

32

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

ERASE

SAMPLE PROBLEM: Suppose

$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

33

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

ERASE

SAMPLE PROBLEM: Suppose

$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

SOLUTION: This easy, just plug everything into the formula given above.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$85 = 50 + 40 + 35 - 18 - 12 - n(A \cap C) + 5$

$n(A \cap C) = 50 + 40 + 35 - 18 - 12 + 5 - 85 = 15$

34

Lecture 2

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

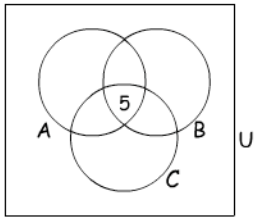
ERASE

SAMPLE PROBLEM: Suppose

$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

ALTERNATE METHOD OF SOLUTION:
Fill out a Venn diagram.



35

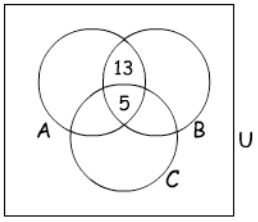
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

ERASE

SAMPLE PROBLEM: Suppose

$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.



36

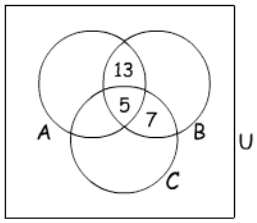
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

ERASE

SAMPLE PROBLEM: Suppose

$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.



37

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

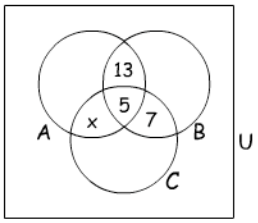
ERASE

SAMPLE PROBLEM: Suppose

$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

NOW YOU ARE STUCK!!!
CALL THE UNKNOWN x
CONTINUE AND SOLVE FOR x.



38

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

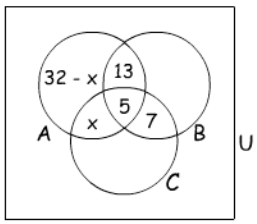
ERASE

SAMPLE PROBLEM: Suppose

$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

NOW YOU ARE STUCK!!!
CALL THE UNKNOWN x
CONTINUE AND SOLVE FOR x.



39

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

ERASE

SAMPLE PROBLEM: Suppose

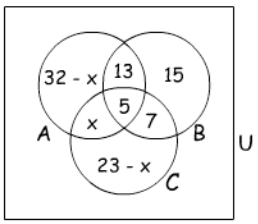
$n(A \cup B \cup C) = 85$
$n(A) = 50 \quad n(B) = 40 \quad n(C) = 35$
$n(A \cap B) = 18 \quad n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

$40 + 23 - x + x + 32 - x = 85$

So $x = 10$.

$n(A \cap C) = 15$



40

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

SAMPLE PROBLEM: Suppose

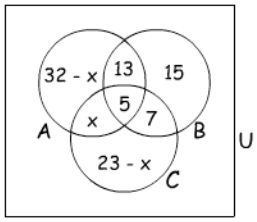
$n(A \cup B \cup C) = 85$
$n(A) = 50$ $n(B) = 40$ $n(C) = 35$
$n(A \cap B) = 18$ $n(B \cap C) = 12$
$n(A \cap B \cap C) = 5$

Find $n(A \cap C)$.

$40 + 23 - x + x + 32 - x = 85$

So $x = 10$.

$n(A \cap C) = 15$



ERASE

41

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

ERASE

42

Lecture 2

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

Let,
U = set consisting of the 110 tourists.
A = set of tourists with Argentine citizenship.
B = set of tourists with Brazilian citizenship.
C = set of tourists with Canadian citizenship.

$n(U) = 110$ $n(A) = 50$ $n(B) = 40$ $n(C) = 35$

ERASE

43

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n(U) = 110$ $n(A) = 50$ $n(B) = 40$ $n(C) = 35$

ERASE

44

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n((A \cup B \cup C)^c) = 25 = 110 - n(A \cup B \cup C) \Rightarrow n(A \cup B \cup C) = 85$

$n(U) = 110$ $n(A) = 50$ $n(B) = 40$ $n(C) = 35$

ERASE

45

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n(U) = 110$ $n(A) = 50$ $n(B) = 40$ $n(C) = 35$ $n(A \cup B \cup C) = 85$

ERASE

46

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n(A \cap B) = 18$

$n(U) = 110$ $n(A) = 50$ $n(B) = 40$ $n(C) = 35$ $n(A \cup B \cup C) = 85$

ERASE

47

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n(A \cap B) = 18$

$n(U) = 110$ $n(A) = 50$ $n(B) = 40$ $n(C) = 35$ $n(A \cup B \cup C) = 85$

ERASE

48

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n(A \cap B) = 18 \quad n(B \cap C) = 12$

$n(U) = 110 \quad n(A) = 50 \quad n(B) = 40 \quad n(C) = 35 \quad n(A \cup B \cup C) = 85$

49

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n(A \cap B) = 18 \quad n(B \cap C) = 12 \quad n(A \cap B \cap C) = 5$

$n(U) = 110 \quad n(A) = 50 \quad n(B) = 40 \quad n(C) = 35 \quad n(A \cup B \cup C) = 85$

50

NEW PROBLEM: Of 110 tourists, 50 are citizens of Argentina, 40 are citizens of Brazil, 35 are Canadian citizens, and 25 are are not citizens of any of these 3 countries. 18 are citizens of both Argentina and Brazil, and 12 are citizens of Brazil and Canada. 5 are citizens of Argentina, Brazil, and Canada. How many are citizens of Argentina and Canada?

STEP 1: SET UP

$n(A \cap C) = ?$

$n(A \cap B) = 18 \quad n(B \cap C) = 12 \quad n(A \cap B \cap C) = 5$

$n(U) = 110 \quad n(A) = 50 \quad n(B) = 40 \quad n(C) = 35 \quad n(A \cup B \cup C) = 85$

51

STEP 2: SOLVE THE MATH PROBLEM

$n(A \cap C) = ?$

$n(A \cap B) = 18 \quad n(B \cap C) = 12 \quad n(A \cap B \cap C) = 5$

$n(U) = 110 \quad n(A) = 50 \quad n(B) = 40 \quad n(C) = 35 \quad n(A \cup B \cup C) = 85$

52

STEP 2: SOLVE THE MATH PROBLEM

$n(A \cap C) = ?$ **DONE: SEE PREVIOUS EXAMPLE**

$n(A \cap B) = 18 \quad n(B \cap C) = 12 \quad n(A \cap B \cap C) = 5$

$n(U) = 110 \quad n(A) = 50 \quad n(B) = 40 \quad n(C) = 35 \quad n(A \cup B \cup C) = 85$

53

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

54

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

STEP 1: SET UP

Let,

U = set consisting of the 100 listeners.

W = listeners in U that want one weather update/hour.

C = listeners in U that want only 3 commercials/hour.

$n(U) = 100 \quad n(W) = 60 \quad n(C) = 45$

55

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

STEP 1: SET UP

"25 people want both" $\Rightarrow n(W \cap C) = 25$

$n(U) = 100 \quad n(W) = 60 \quad n(C) = 45$

56

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

STEP 1: SET UP

"25 people want both" ⇒

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

57

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

STEP 1: SET UP

don't want both a weather update/hour and don't want 3 commercials/hr.

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

58

Lecture 2

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

STEP 1: SET UP

don't want both a weather update/hour and don't want 3 commercials/hr.

W^c ? C^c

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

59

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

STEP 1: SET UP

don't want both a weather update/hour and don't want 3 commercials/hr.

W^c $\cap$ C^c

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

60

NEW PROBLEM: Of 100 listeners of radio station, 60 prefer to have weather updates once an hour, 45 want 3 commercials an hour (as opposed to sporadically throughout), and 25 people want both (1 weather update/hour and 3 commercials/hour). How many don't want both a weather update/hour and don't want 3 commercials/hour?

STEP 1: SET UP

don't want both a weather update/hour and don't want 3 commercials/hr.

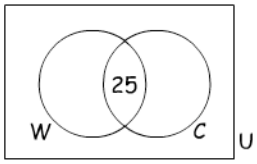
W^c $\cap$ C^c

$n(W^c \cap C^c) = ?$

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

61

STEP 2: SOLVE THE MATH PROBLEM

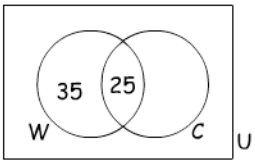


$n(W^c \cap C^c) = ?$

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

62

STEP 2: SOLVE THE MATH PROBLEM

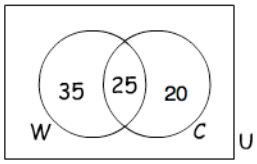


$n(W^c \cap C^c) = ?$

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

63

STEP 2: SOLVE THE MATH PROBLEM

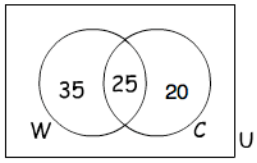


$n(W^c \cap C^c) = ?$

$n(U) = 100$ $n(W) = 60$ $n(C) = 45$ $n(W \cap C) = 25$

64

STEP 2: SOLVE THE MATH PROBLEM



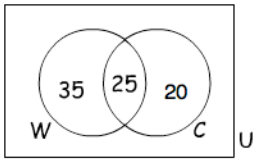
$\Rightarrow n(W \cup C) = 35 + 25 + 20 = 80$

$n(W^c \cap C^c) = ?$

$n(U) = 100 \quad n(W) = 60 \quad n(C) = 45 \quad n(W \cap C) = 25$

65

STEP 2: SOLVE THE MATH PROBLEM



$\Rightarrow n(W \cup C) = 35 + 25 + 20 = 80$

$\Rightarrow n((W \cup C)^c) = n(U) - n(W \cup C)$
$= 100 - 80 = 20$

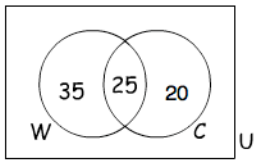
$n(W^c \cap C^c) = ?$

$n(U) = 100 \quad n(W) = 60 \quad n(C) = 45 \quad n(W \cap C) = 25$

66

Lecture 2

STEP 2: SOLVE THE MATH PROBLEM



$\Rightarrow n(W \cup C) = 35 + 25 + 20 = 80$

$\Rightarrow n((W \cup C)^c) = n(U) - n(W \cup C)$

$\Rightarrow n((W \cup C)^c) = 100 - 80 = 20$

$\Rightarrow n(W^c \cap C^c) = 20$

DONE!!!

$n(W^c \cap C^c) = ?$

$n(U) = 100 \quad n(W) = 60 \quad n(C) = 45 \quad n(W \cap C) = 25$

67

STEP 2: ALTERNATE WAY TO SOLVE MATH PROBLEM

$n(W^c \cap C^c) = n((W \cup C)^c) = n(U) - n(W \cup C)$
$= n(U) - (n(W) + n(C) - n(W \cap C))$
$= 100 - (60 + 45 - 25) = 100 - 80 = 20$

DONE!!!

$n(W^c \cap C^c) = ?$

$n(U) = 100 \quad n(W) = 60 \quad n(C) = 45 \quad n(W \cap C) = 25$

68

NEW PROBLEM: Of 100 College students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

69

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP

Let,
U = set consisting of the 100 college students.
C = subset of U consisting of those students that drive a car.
T = subset consisting of those students that ride the bus.
B = subset consisting of those students that ride a bike.

$n(U) = 100 \quad n(C) = 8 \quad n(T) = 20 \quad n(B) = 48$

70

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP

$n((C \cup T \cup B)^c) = 38 \Rightarrow n(C \cup T \cup B) = 100 - 38 = 62.$ ($n(U)$)

$n(U) = 100 \quad n(C) = 8 \quad n(T) = 20 \quad n(B) = 48$

71

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP

$n((C \cup T \cup B)^c) = 38 \Rightarrow n(C \cup T \cup B) = 100 - 38 = 62.$ ($n(U)$)

What about the last sentence?

$n(U) = 100 \quad n(C) = 8 \quad n(T) = 20 \quad n(B) = 48$

72

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP $n(U)$

$n((C \cup T \cup B)^c) = 38 \Rightarrow n(C \cup T \cup B) = 100 - 38 = 62.$

What about the last sentence? $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

73

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP What are they asking for?

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

74

Lecture 2

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP What are they asking for?

"ride a bike and also drive a car or take the bus"
B C T

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

75

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP What are they asking for?

"ride a bike and (also drive a car or take the bus)"
B $\cap$ (C $\cup$ T)

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

76

NEW PROBLEM: Of 100 college students, 8 drive a car, 20 use public transportation (i.e. the bus), 48 ride a bicycle, 38 use none of these modes of transportation. Additionally, no student that drives a car takes the bus.
How many ride a bike and also drive a car or take the bus?

STEP 1: SET UP What are they asking for?

"ride a bike and (also drive a car or take the bus)"
B $\cap$ (C $\cup$ T)

$n(B \cap (C \cup T)) = ?$

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

77

STEP 2: SOLVE

$n(T \cup B \cup C)$
$= n(T) + n(B) + n(C) - n(T \cap B) - n(B \cap C) - n(T \cap C) + n(T \cap B \cap C)$

$n(B \cap (C \cup T)) = ?$

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

78

STEP 2: SOLVE

$n(T \cup B \cup C)$
$= n(T) + n(B) + n(C) - n(T \cap B) - n(B \cap C) - n(T \cap C) + n(T \cap B \cap C)$
$62 = 20 + 48 + 8 - n(T \cap B) - n(B \cap C) - 0 + n(T \cap B \cap C)$

$n(B \cap (C \cup T)) = ?$

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

79

STEP 2: SOLVE

$n(T \cup B \cup C)$
$= n(T) + n(B) + n(C) - n(T \cap B) - n(B \cap C) - n(T \cap C) + n(T \cap B \cap C)$
$62 = 20 + 48 + 8 - n(T \cap B) - n(B \cap C) - 0 + n(T \cap B \cap C)$
$62 = 20 + 48 + 8 - (n(T \cap B) + n(B \cap C)) - 0 + 0$

$n(B \cap (C \cup T)) = ?$

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

80

STEP 2: SOLVE

$n(T \cup B \cup C)$

$= n(T) + n(B) + n(C) - n(T \cap B) - n(B \cap C) - n(T \cap C) + n(T \cap B \cap C)$

$62 = 20 + 48 + 8 - n(T \cap B) - n(B \cap C) - 0 + n(T \cap B \cap C)$

$62 = 20 + 48 + 8 - (n(T \cap B) + n(B \cap C)) - 0 + 0$

TRICK: $T \cap B$ AND $B \cap C$ ARE DISJOINT, SO

$n(T \cap B) + n(B \cap C) = n((T \cap B) \cup (B \cap C)) = n(B \cap (T \cup C))$

$n(B \cap (C \cup T)) = ?$

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

81

STEP 2: SOLVE

$n(T \cup B \cup C)$

$= n(T) + n(B) + n(C) - n(T \cap B) - n(B \cap C) - n(T \cap C) + n(T \cap B \cap C)$

$62 = 20 + 48 + 8 - n(T \cap B) - n(B \cap C) - 0 + n(T \cap B \cap C)$

$62 = 20 + 48 + 8 - (n(T \cap B) + n(B \cap C)) - 0 + 0$

$62 = 20 + 48 + 8 - n(B \cap (C \cup T)) - 0 + 0$

$n(B \cap (C \cup T)) = ?$

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

82

Lecture 2

STEP 2: SOLVE

$n(T \cup B \cup C)$

$= n(T) + n(B) + n(C) - n(T \cap B) - n(B \cap C) - n(T \cap C) + n(T \cap B \cap C)$

$62 = 20 + 48 + 8 - n(T \cap B) - n(B \cap C) - 0 + n(T \cap B \cap C)$

$62 = 20 + 48 + 8 - (n(T \cap B) + n(B \cap C)) - 0 + 0$

$62 = 20 + 48 + 8 - n(B \cap (C \cup T)) - 0 + 0$

$62 = 76 - n(B \cap (C \cup T)) \Rightarrow \mathbf{n(B \cap (C \cup T)) = 14}$

DONE!!!

$n(B \cap (C \cup T)) = ?$

$n(C \cup T \cup B) = 62.$ $n(C \cap T) = 0.$

$n(U) = 100$ $n(C) = 8$ $n(T) = 20$ $n(B) = 48$

83