

**PROBLEM:** William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

ERASE



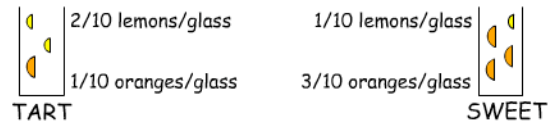
1

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Lecture 16

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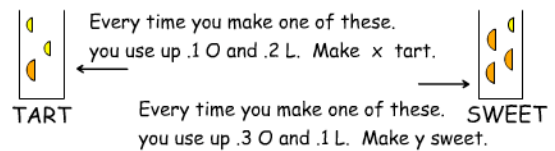
3

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How many oranges do you use?

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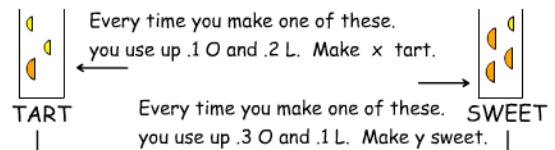
5

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$.1x$

# of sweet times number =  $.3y$   
of O per sweet drink

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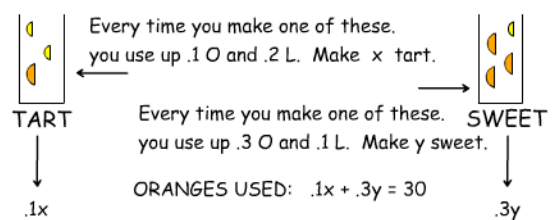
7

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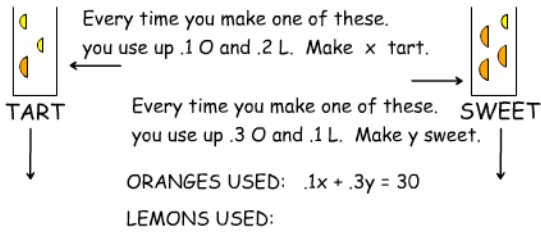


$.1x$

ORANGES USED:  $.1x + .3y = 30$

$.3y$

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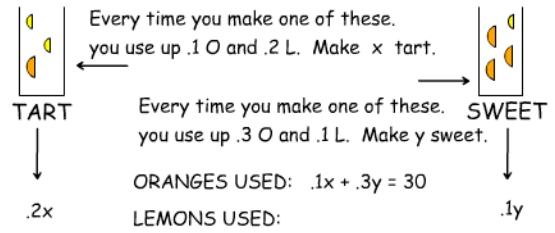
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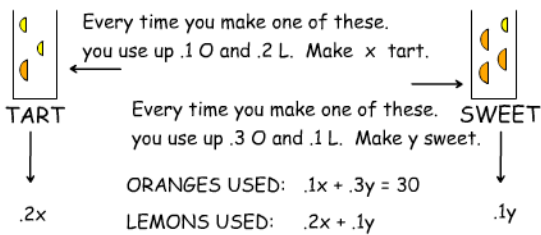


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Lecture 16

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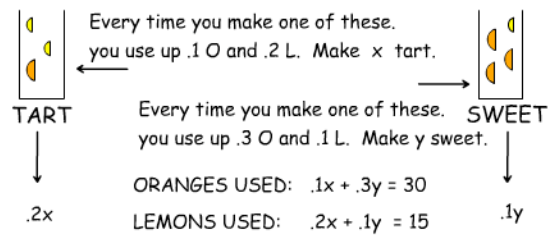
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$$.1x + .3y = 30$$

$$.2x + .1y = 15$$

ERASE



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$$.2x + .6y = 60$$

$$.2x + .1y = 15$$

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**PROBLEM:** William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$.2x + .6y = 60$$

$$-(.2x + .1y = 15)$$

ERASE



16



**PROBLEM:** William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$\begin{array}{r} .2x + .6y = 60 \\ -(.2x + .1y = 15) \\ \hline .5y = 45 \end{array}$$

ERASE



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$$\begin{array}{r} .2x + .6y = 60 \\ -(.2x + .1y = 15) \\ \hline .5y = 45 \end{array} \quad y = 90$$

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Lecture 16

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$$\begin{array}{r} .1x + .3y = 30 \\ .2x + .1y = 15 \end{array} \quad y = 90$$

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$$\begin{array}{r} .1x + .3y = 30 \\ -(.6x + .3y = 45) \\ \hline \end{array} \quad y = 90$$

ERASE



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$$\begin{array}{r} .1x + .3y = 30 \\ -(.6x + .3y = 45) \\ \hline \end{array} \quad y = 90$$

$$\begin{array}{r} -.5x = -15 \end{array}$$

ERASE



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ERASE



22



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$$\begin{array}{r} .1x + .3y = 30 \\ -(.6x + .3y = 45) \\ \hline \end{array} \quad y = 90$$

$$\begin{array}{r} -.5x = -15 \end{array} \quad x = 30$$

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Make 30 tart and 90 sweet drinks.

Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

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Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$2 \cdot x_1 + 3 \cdot x_2 - 3 \cdot x_3 = 8$$

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Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$\begin{matrix} 1 & 2 & 0 \\ 2 \cdot x_1 + 3 \cdot x_2 - 3 \cdot x_3 = 8 \end{matrix}$$

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Lecture 16

Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$2 \cdot 1 + 3 \cdot 2 - 3 \cdot 0 = 8$$

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Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

Now multiply both sides by 2:  $2 \cdot 1 + 3 \cdot 2 - 3 \cdot 0 = 8$

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Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$4 \cdot 1 + 6 \cdot 2 - 6 \cdot 0 = 16$$

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Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$4 \cdot 1 + 6 \cdot 2 - 6 \cdot 0 = 16$$

So  $x_1 = 1, x_2 = 2, x_3 = 0$  is a solution of

$$4 \cdot x_1 + 6 \cdot x_2 - 6 \cdot x_3 = 16$$

The equation obtained by multiplying all coefficients in the original equation by 2.

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Here is an equation in 3 variables  $x_1, x_2, x_3$ .

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

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The equation obtained by multiplying all coefficients in the original equation by 2.

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Conclusion: Given an equation and a solution, any "multiple" of that equation also has the same solution (and maybe others).

$$2x_1 + 3x_2 - 3x_3 = 8$$

$$2kx_1 + 3kx_2 - 3kx_3 = 8k$$

$k = \text{constant}$

Any solution of this equation is a solution of this equation.

$x_1 = 1, x_2 = 2, x_3 = 0$  is a solution of both equations.

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Now consider two equations:

$$\begin{aligned} 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - x_3 &= 5 \end{aligned}$$

One solution to both equations is  $x_1 = 1, x_2 = 2, x_3 = 0$ .

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Now consider two equations:

$$\begin{aligned} 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

One solution to both equations is  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$\begin{aligned} 2 \times \text{Eq. 1)} & \quad 4 \cdot x_1 + 6 \cdot x_2 - 6 \cdot x_3 = 16 \\ \text{Eq. 2)} & \quad 1 \cdot x_1 + 2 \cdot x_2 - 1 \cdot x_3 = 5 \end{aligned}$$

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Lecture 16

Now consider two equations:

$$\begin{aligned} 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

One solution to both equations is  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$\begin{aligned} 2 \times \text{Eq. 1)} & \quad 4 \cdot 1 + 6 \cdot 2 - 6 \cdot 0 = 16 \\ \text{Eq. 2)} & \quad 1 \cdot 1 + 2 \cdot 2 - 1 \cdot 0 = 5 \end{aligned}$$

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Now consider two equations:

$$\begin{aligned} 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

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$$\begin{aligned} 2 \times \text{Eq. 1)} & \quad 4 \cdot 1 + 6 \cdot 2 - 6 \cdot 0 = 16 \\ \text{Eq. 2)} & \quad + \quad 1 \cdot 1 + 2 \cdot 2 - 1 \cdot 0 = 5 \end{aligned}$$

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Now consider two equations:

$$\begin{aligned} 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

One solution to both equations is  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$\begin{aligned} 2 \times \text{Eq. 1)} & \quad 4 \cdot 1 + 6 \cdot 2 - 6 \cdot 0 = 16 \\ \text{Eq. 2)} & \quad + \quad 1 \cdot 1 + 2 \cdot 2 - 1 \cdot 0 = 5 \\ \hline & \quad 5 \cdot 1 + 8 \cdot 2 - 7 \cdot 0 = 21 \end{aligned}$$

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Now consider two equations:

$$\begin{aligned} 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

One solution to both equations is  $x_1 = 1, x_2 = 2, x_3 = 0$ .

$$\begin{aligned} 2 \times \text{Eq. 1)} & \quad 4 \cdot 1 + 6 \cdot 2 - 6 \cdot 0 = 16 \\ \text{Eq. 2)} & \quad 1 \cdot 1 + 2 \cdot 2 - 1 \cdot 0 = 5 \\ \hline & \quad 5 \cdot 1 + 8 \cdot 2 - 7 \cdot 0 = 21 \end{aligned}$$

So  $x_1 = 1, x_2 = 2, x_3 = 0$  is a solution of

$$5x_1 + 8x_2 - 7x_3 = 21$$

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Now consider two equations:

$$\begin{aligned} 2 \times [2x_1 + 3x_2 - 3x_3 &= 8] \\ + 1x_1 + 2x_2 - 1x_3 &= 5 \\ \hline 5x_1 + 8x_2 - 7x_3 &= 21 \end{aligned}$$

Conclusion: A multiple of one equation added to another yields a new equation, and any solution of the original equations is a solution of the new equation.

So  $x_1 = 1, x_2 = 2, x_3 = 0$  is a solution of

$$5x_1 + 8x_2 - 7x_3 = 21$$

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Here's 3 equations and 3 unknowns:

$$\begin{aligned} x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - x_3 &= 5 \end{aligned}$$

Again  $x_1 = 1, x_2 = 2, x_3 = 0$  is a solution of these 3 equations.

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Here's 3 equations and 3 unknowns:

$$\begin{aligned} 0x_1 + 1x_2 + 1x_3 &= 2 \\ 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

This system has a solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .  
 There are 3 types of row operations that will yield a new system that also has solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

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Here's 3 equations and 3 unknowns:

$$\begin{aligned} 0x_1 + 1x_2 + 1x_3 &= 2 \\ 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

Basic operations:

- 1) Interchange 2 rows (i.e. equations).
- 2) Multiply one equation by a number ( $\neq 0$ ).
- 3) Add a multiple of one equation to another.

Any solution to the original system is a solution to the "new" system.

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Lecture 16

More streamlined notation:

$$\begin{aligned} 0x_1 + 1x_2 + 1x_3 &= 2 \\ 2x_1 + 3x_2 - 3x_3 &= 8 \\ 1x_1 + 2x_2 - 1x_3 &= 5 \end{aligned}$$

Basic operations:

- 1) Interchange 2 rows (i.e. equations).
- 2) Multiply one equation by a number ( $\neq 0$ ).
- 3) Add a multiple of one equation to another.

Any solution to the original system is a solution to the "new" system.

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More streamlined notation:

$$\begin{array}{cccc} 0 & 1 & 1 & 2 \\ 2 & 3 & -3 & 8 \\ 1 & 2 & -1 & 5 \end{array}$$

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More streamlined notation:

$$\begin{array}{cccc} 0 & 1 & 1 & 2 \\ 2 & 3 & -3 & 8 \\ 1 & 2 & -1 & 5 \end{array}$$

Any system can be trimmed down to the coefficients and reconstituted later by putting the + signs and = signs back in. Here's 2 x 2nd row added to 3rd row:

$$\begin{array}{r} 2 \times [2x_1 + 3x_2 - 3x_3 = 8] \\ + 1x_1 + 2x_2 - 1x_3 = 5 \\ \hline 5x_1 + 8x_2 - 7x_3 = 21 \end{array}$$

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More streamlined notation:

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Any system can be trimmed down to the coefficients and reconstituted later by putting the + signs and = signs back in. Here's 2 x 2nd row added to 3rd row:

$$\begin{array}{r} 2 \times [2 \quad 3 \quad -3 \quad 8] \\ + 1 \quad 2 \quad -1 \quad 5 \\ \hline 5 \quad 8 \quad -7 \quad 21 \end{array}$$

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More streamlined notation:

$$\begin{array}{cccc} 0 & 1 & 1 & 2 \\ 2 & 3 & -3 & 8 \\ \text{Throw away} & & & \\ \text{row 3} \longrightarrow & 1 & 2 & -1 & 5 \end{array}$$

$$\begin{array}{cccc} 5 & 8 & -7 & 21 \end{array}$$

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More streamlined notation:

$$\begin{array}{cccc} 0 & 1 & 1 & 2 \\ 2 & 3 & -3 & 8 \\ & & & \end{array}$$

$$\begin{array}{cccc} 5 & 8 & -7 & 21 \end{array}$$

ERASE



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More streamlined notation:

$$\begin{array}{cccc} 0 & 1 & 1 & 2 \\ 2 & 3 & -3 & 8 \\ 5 & 8 & -7 & 21 \end{array}$$

ERASE



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More streamlined notation:

$$\begin{array}{cccc} 0 & 1 & 1 & 2 \\ 2 & 3 & -3 & 8 \\ 5 & 8 & -7 & 21 \end{array}$$

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Lecture 16

More streamlined notation:

$$\begin{array}{l} 0x_1 + 1x_2 + 1x_3 = 2 \\ 2x_1 + 3x_2 - 3x_3 = 8 \\ 5x_1 + 8x_2 - 7x_3 = 21 \end{array}$$

Here is a new system of equations that still has solution  $x_1 = 1, x_2 = 2, x_3 = 0$ . Interchanging 2 rows, would yield a new system with this same solution as well. Also multiplication of a single row by a number would yield a new system that has solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

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More streamlined notation:

$$\begin{array}{cccc} 0 & 1 & 1 & 2 \\ 2 & 3 & -3 & 8 \\ 5 & 8 & -7 & 21 \end{array}$$

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More streamlined notation:

$$\begin{array}{cccc} 0 & 3 & 3 & 6 \\ 2 & 3 & -3 & 8 \\ 5 & 8 & -7 & 21 \end{array}$$

Here is a new system of equations that still has solution  $x_1 = 1, x_2 = 2, x_3 = 0$ . Interchanging 2 rows, would yield a new system with this same solution as well. Also multiplication of a single row by a number would yield a new system that has solution  $x_1 = 1, x_2 = 2, x_3 = 0$ .

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WHY  
DO THIS??

ERASE



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WHY  
DO THIS??

The object is to take a complicated array of numbers (i.e. system of equations) and make it look simpler.

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