PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

1

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

TART: 2/10 lemons/glass, 1/10 oranges/glass

SWEET: 1/10 lemons/glass, 3/10 oranges/glass

2

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

TART: Every time you make one of these. you use up .1 O and .2 L. Make x tart.

SWEET: Every time you make one of these. you use up .3 O and .1 L. Make y sweet.

3

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

TART: Every time you make one of these. you use up .1 O and .2 L. Make x tart.

SWEET: Every time you make one of these. you use up .3 O and .1 L. Make y sweet.

How many oranges do you use?

4

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

TART: Every time you make one of these. you use up .1 O and .2 L. Make x tart.

SWEET: Every time you make one of these. you use up .3 O and .1 L. Make y sweet.

.1x = # of tart drinks times (number of O) per tart drink

5

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

TART: Every time you make one of these. you use up .1 O and .2 L. Make x tart.

SWEET: Every time you make one of these. you use up .3 O and .1 L. Make y sweet.

.1x

# of sweet times number of O per sweet drink = .3y

6

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

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.1x

ORANGES USED: .1x + .3y

.3y

7

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

TART: Every time you make one of these. you use up .1 O and .2 L. Make x tart.

SWEET: Every time you make one of these. you use up .3 O and .1 L. Make y sweet.

.1x

ORANGES USED: .1x + .3y = 30

.3y

8

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

Every time you make one of these, you use up .1 O and .2 L. Make x tart.

TART

Every time you make one of these, you use up .3 O and .1 L. Make y sweet.

SWEET

ORANGES USED: $.1x + .3y = 30$

LEMONS USED:

9

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

Every time you make one of these, you use up .1 O and .2 L. Make x tart.

TART

Every time you make one of these, you use up .3 O and .1 L. Make y sweet.

SWEET

$.2x$

ORANGES USED: $.1x + .3y = 30$

LEMONS USED:

$.1y$

10

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

Every time you make one of these, you use up .1 O and .2 L. Make x tart.

TART

Every time you make one of these, you use up .3 O and .1 L. Make y sweet.

SWEET

$.2x$

ORANGES USED: $.1x + .3y = 30$

LEMONS USED: $.2x + .1y$

$.1y$

11

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

Every time you make one of these, you use up .1 O and .2 L. Make x tart.

TART

Every time you make one of these, you use up .3 O and .1 L. Make y sweet.

SWEET

$.2x$

ORANGES USED: $.1x + .3y = 30$

LEMONS USED: $.2x + .1y = 15$

$.1y$

12

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$.1x + .3y = 30$

$.2x + .1y = 15$

13

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$.1x + .3y = 30$

$.2x + .1y = 15$

14

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$.2x + .6y = 60$

$.2x + .1y = 15$

15

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$.2x + .6y = 60$

$-(.2x + .1y = 15)$

16

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$
\begin{array}{r}
.2x + .6y = 60 \\
-(.2x + .1y = 15) \\
\hline
.5y = 45
\end{array}
$$

17

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$
\begin{array}{r}
.2x + .6y = 60 \\
-(.2x + .1y = 15) \\
\hline
.5y = 45
\end{array}
$$

$y = 90$

18

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$
\begin{array}{r}
.1x + .3y = 30 \\
.2x + .1y = 15
\end{array}
$$

$y = 90$

19

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$
\begin{array}{r}
.1x + .3y = 30 \\
-(.6x + .3y = 45)
\end{array}
$$

$y = 90$

20

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$
\begin{array}{r}
.1x + .3y = 30 \\
-(.6x + .3y = 45) \\
\hline
-.5x \quad = -15
\end{array}
$$

$y = 90$

21

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$
\begin{array}{r}
.1x + .3y = 30 \\
-(.6x + .3y = 45) \\
\hline
-.5x \quad = -15
\end{array}
$$

$y = 90$

$x = 30$

22

PROBLEM: William and Margaret have 15 lemons and 30 oranges. They use 2 lemons and 1 orange to make 10 glasses of tart drink, and 1 lemon and 3 oranges to make 10 glasses of sweet drink. How many tart and how many sweet glasses should they make to use up all of their oranges and lemons?

$$
\begin{array}{r}
.1x + .3y = 30 \\
-(.6x + .3y = 45) \\
\hline
-.5x \quad = -15
\end{array}
$$

$y = 90$

$x = 30$

Make 30 tart and 90 sweet drinks.

23

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

24

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

$$2\cdot x_1 + 3\cdot x_2 - 3\cdot x_3 = 8$$

25

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

$$\overset{1}{2\cdot x_1} + \overset{2}{3\cdot x_2} - \overset{0}{3\cdot x_3} = 8$$

26

Lecture 16

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

$$2\cdot 1 + 3\cdot 2 - 3\cdot 0 = 8$$

27

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

Now multiply both sides by 2: $2\cdot 1 + 3\cdot 2 - 3\cdot 0 = 8$

28

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

$$4\cdot 1 + 6\cdot 2 - 6\cdot 0 = 16$$

29

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

One solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

$$4\cdot 1 + 6\cdot 2 - 6\cdot 0 = 16$$

So $x_1 = 1$, $x_2 = 2$, $x_3 = 0$ is a solution of

$$4\cdot x_1 + 6\cdot x_2 - 6\cdot x_3 = 16$$

The equation obtained by multiplying all coefficients in the original equation by 2.

30

Here is an equation in 3 variables x_1, x_2, x_3.

$$2x_1 + 3x_2 - 3x_3 = 8$$

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$$4\cdot x_1 + 6\cdot x_2 - 6\cdot x_3 = 16$$

The equation obtained by multiplying all coefficients in the original equation by 2.

31

Conclusion: Given an equation and a solution, any "multiple" of that equation also has the same solution (and maybe others).

$$2x_1 + 3x_2 - 3x_3 = 8 \qquad 2k\,x_1 + 3k\,x_2 - 3k\,x_3 = 8k$$

k = constant

Any solution of this equation is a solution of this equation.

$x_1 = 1$, $x_2 = 2$, $x_3 = 0$ is a solution of both equations.

32

Now consider two equations:

$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - x_3 = 5$$

One solution to both equations is $x_1 = 1,\ x_2 = 2,\ x_3 = 0$.

Now consider two equations:

$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - 1x_3 = 5$$

One solution to both equations is $x_1 = 1,\ x_2 = 2,\ x_3 = 0$.

2 x Eq. 1) $\quad 4\cdot x_1 + 6\cdot x_2 - 6\cdot x_3 = 16$

Eq. 2) $\quad 1\cdot x_1 + 2\cdot x_2 - 1\cdot x_3 = 5$

Now consider two equations:

$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - 1x_3 = 5$$

One solution to both equations is $x_1 = 1,\ x_2 = 2,\ x_3 = 0$.

2 x Eq. 1) $\quad 4\cdot 1 + 6\cdot 2 - 6\cdot 0 = 16$

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Now consider two equations:

$$2x_1 + 3x_2 - 3x_3 = 8$$
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One solution to both equations is $x_1 = 1,\ x_2 = 2,\ x_3 = 0$.

2 x Eq. 1) $\quad 4\cdot 1 + 6\cdot 2 - 6\cdot 0 = 16$

Eq. 2) $\quad + \ 1\cdot 1 + 2\cdot 2 - 1\cdot 0 = 5$

Now consider two equations:

$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - 1x_3 = 5$$

One solution to both equations is $x_1 = 1,\ x_2 = 2,\ x_3 = 0$.

2 x Eq. 1) $\quad 4\cdot 1 + 6\cdot 2 - 6\cdot 0 = 16$

Eq. 2) $\quad + \ 1\cdot 1 + 2\cdot 2 - 1\cdot 0 = 5$

$\quad 5\cdot 1 + 8\cdot 2 - 7\cdot 0 = 21$

Now consider two equations:

$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - 1x_3 = 5$$

One solution to both equations is $x_1 = 1,\ x_2 = 2,\ x_3 = 0$.

2 x Eq. 1) $\quad 4\cdot 1 + 6\cdot 2 - 6\cdot 0 = 16$

Eq. 2) $\quad 1\cdot 1 + 2\cdot 2 - 1\cdot 0 = 5$

$\quad 5\cdot 1 + 8\cdot 2 - 7\cdot 0 = 21$

So $x_1 = 1,\ x_2 = 2,\ x_3 = 0$. is a solution of

$$5x_1 + 8x_2 - 7x_3 = 21$$

Now consider two equations:

$$2 \times [2x_1 + 3x_2 - 3x_3 = 8]$$
$$+ \ 1x_1 + 2x_2 - 1x_3 = 5$$
$$5x_1 + 8x_2 - 7x_3 = 21$$

> Conclusion: A multiple of one equation added to another yields a new equation, and any solution of the original equations is a solution of the new equation.

So $x_1 = 1,\ x_2 = 2,\ x_3 = 0$. is a solution of

$$5x_1 + 8x_2 - 7x_3 = 21$$

Here's 3 equations and 3 unknowns:

$$x_2 + x_3 = 2$$
$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - x_3 = 5$$

Again $x_1 = 1,\ x_2 = 2,\ x_3 = 0$. is a solution of these 3 equations.

Here's 3 equations and 3 unknowns:

$$0x_1 + 1x_2 + 1x_3 = 2$$
$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - 1x_3 = 5$$

This system has a solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$. There are 3 types of row operations that will yield a new system that also has solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

Here's 3 equations and 3 unknowns:

$$0x_1 + 1x_2 + 1x_3 = 2$$
$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - 1x_3 = 5$$

Basic operations:

1) Interchange 2 rows (i.e. equations).
2) Multiply one equation by a number ($\neq 0$).
3) Add a multiple of one equation to another.

Any solution to the original system is a solution to the "new" system.

More streamlined notation:

$$0x_1 + 1x_2 + 1x_3 = 2$$
$$2x_1 + 3x_2 - 3x_3 = 8$$
$$1x_1 + 2x_2 - 1x_3 = 5$$

Basic operations:

1) Interchange 2 rows (i.e. equations).
2) Multiply one equation by a number ($\neq 0$).
3) Add a multiple of one equation to another.

Any solution to the original system is a solution to the "new" system.

More streamlined notation:

0	1	1	2
2	3	-3	8
1	2	-1	5

More streamlined notation:

0	1	1	2
2	3	-3	8
1	2	-1	5

Any system can be trimmed down to the coefficients and reconstituted later by putting the + signs and = signs back in. Here's 2 x 2nd row added to 3rd row:

$$2 \times [2x_1 + 3x_2 - 3x_3 = 8]$$
$$+ \; 1x_1 + 2x_2 - 1x_3 = 5$$
$$5x_1 + 8x_2 - 7x_3 = 21$$

More streamlined notation:

0	1	1	2
2	3	-3	8
1	2	-1	5

Any system can be trimmed down to the coefficients and reconstituted later by putting the + signs and = signs back in. Here's 2 x 2nd row added to 3rd row:

2 x [2	3	-3	8]
+ 1	2	-1	5
5	8	-7	21

More streamlined notation:

	0	1	1	2
	2	3	-3	8
Throw away row 3 →	1	2	-1	5
	5	8	-7	21

More streamlined notation:

0	1	1	2
2	3	-3	8
5	8	-7	21

More streamlined notation:

0	1	1	2
2	3	-3	8
5	8	-7	21

More streamlined notation:

0	1	1	2
2	3	-3	8
5	8	-7	21

More streamlined notation:

$$0x_1 + 1x_2 + 1x_3 = 2$$
$$2x_1 + 3x_2 - 3x_3 = 8$$
$$5x_1 + 8x_2 - 7x_3 = 21$$

Here is a new system of equations that still has solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$. Interchanging 2 rows, would yield a new system with this same solution as well. Also multiplication of a single row by a number would yield a new system that has solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

More streamlined notation:

0	1	1	2
2	3	-3	8
5	8	-7	21

Here is a new system of equations that still has solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$. Interchanging 2 rows, would yield a new system with this same solution as well. Also multiplication of a single row by a number would yield a new system that has solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

More streamlined notation:

0	3	3	6
2	3	-3	8
5	8	-7	21

Here is a new system of equations that still has solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$. Interchanging 2 rows, would yield a new system with this same solution as well. Also multiplication of a single row by a number would yield a new system that has solution $x_1 = 1$, $x_2 = 2$, $x_3 = 0$.

WHY DO THIS??

WHY DO THIS??

The object is to take a complicated array of numbers (i.e. system of equations) and make it look simpler.