RANDOM VARIABLES:

A random variable on a sample space S is an assignment of numbers to the elements of S. Each element of S has one number assigned to it by the random variable.

A random variable is a function on a sample space.

1

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

Comments:

1) Notation for elements in S: (G,R) indicates, for example, that a green was selected then a red.

2) You are being told specifically what S is. In previous problems it was your choice. Here it is not.

2

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

Now define a function X on S by the following:

X(R,R) = 20
X(R,G) = 30
X(G,R) = 30
X(G,G) = 40

Notice that this is how much money you would get (in $) if the red slips were replaced by $10 bills and the green by $20 bills.

3

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

NOTATION:

(X= 20) stands for the event that after the procedure is run, the value of X is 20.

(X= 20) = {(R,R)} $\subset$ S

4

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

NOTATION:

(X= 20) = {(R,R)} $\subset$ S

5

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

NOTATION:

(X= 20) = {(R,R)} $\subset$ S
(X= 30) = {(R,G), (G,R)} $\subset$ S
(X= 40) = {(G,G)} $\subset$ S

the set of outcomes for which X has the value 40

6

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

7

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] = Pr[{(R,R)}]

8

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] = Pr[{(R,R)}]

$$= \frac{C(3,2)}{C(5,2)} = 3/10$$

9

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

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(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] = 3/10

10

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] = 3/10

$$Pr[(X=30)] = Pr[\{(R,G),(G,R)\}] = \frac{3\ 2}{C(5,2)} = 6/10$$

11

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10

12

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

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(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] =

13

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

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(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

14

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

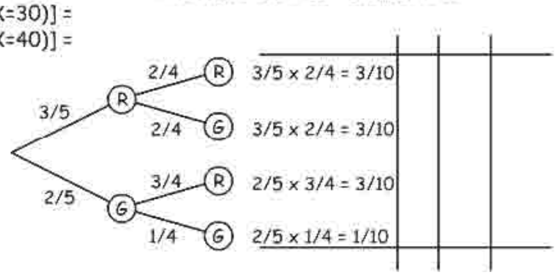
X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM: If the slips are drawn at random, find

Pr[(X=20)] =
Pr[(X=30)] =
Pr[(X=40)] =

Alternate Method: Draw a tree



15

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X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

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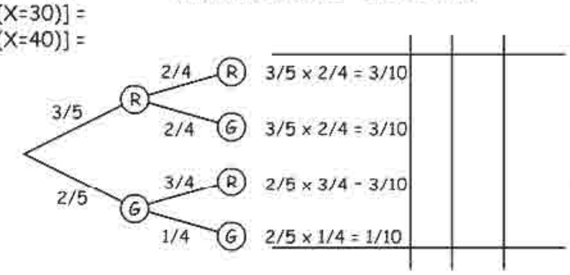
PROBLEM:

Pr[(X=20)] =
Pr[(X=30)] =
Pr[(X=40)] =

Alternate Method: Draw a tree



16

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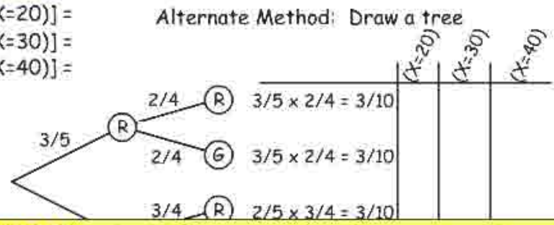
X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM:

Pr[(X=20)] =
Pr[(X=30)] =
Pr[(X=40)] =

Alternate Method: Draw a tree



17

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X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

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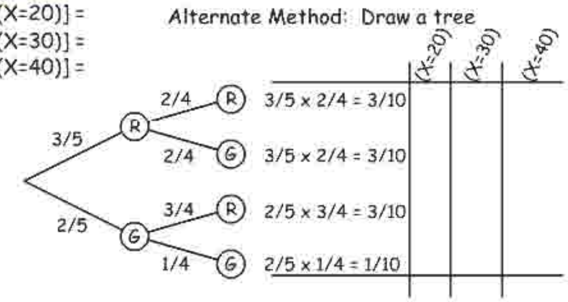
PROBLEM:

Pr[(X=20)] =
Pr[(X=30)] =
Pr[(X=40)] =

Alternate Method: Draw a tree



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Lecture 13

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

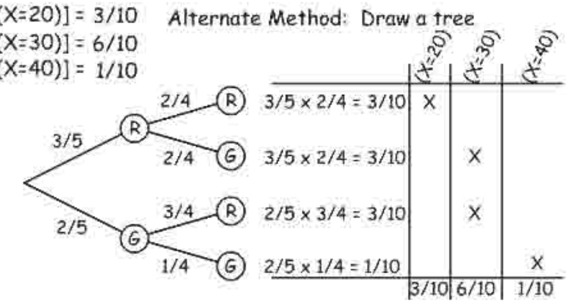
X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

PROBLEM:

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

Alternate Method: Draw a tree



21

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
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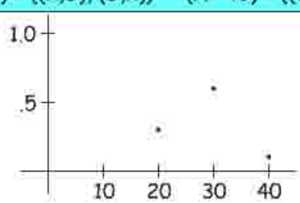
22

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10



GRAPH

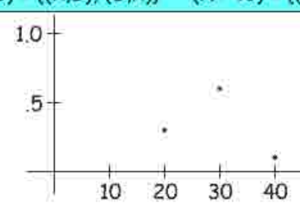
23

EXAMPLE: A hat holds 2 green slips of paper, G_1 and G_2, and 3 red slips, R_1, R_2, R_3. Two slips are removed, one after the other, without replacement. The color and order are recorded and nothing else. Set S = {(R,R), (R,G), (G,R), (G,G)}.

X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10



GRAPH

23

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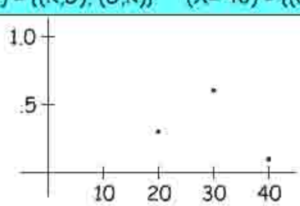
X(R,R) = 20 X(R,G) = 30 X(G,R) = 30 X(G,G) = 40

(X= 20) = {(R,R)} (X= 30) = {(R,G), (G,R)} (X= 40) = {(G,G)}

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10



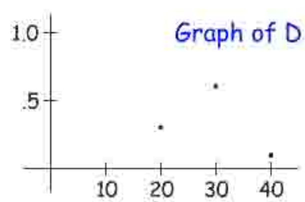
GRAPH

23

This is the graph of a function D. The values that go into D are 20, 30, and 40. The values that come out are the probabilities that X will take on the value 20, or 30, or 40. D is called the probability density function for X.

D(20) = .3 D(30) = .6 D(40) = .1

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10



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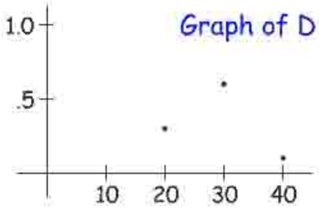
In general, if X takes on the values $x_1, x_2, \ldots x_p$, then

$D(x_i)$ = probability of getting an outcome s whose X value is x_i (i.e. $X(s) = x_i$).

D(20) = .3 D(30) = .6 D(40) = .1

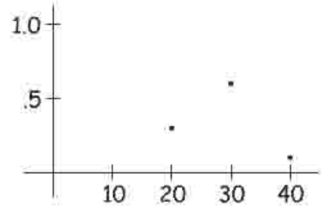
Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10



25

EXAMPLE: If you were to repeat this process repeatedly*, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial? *many, many times

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10



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EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win $20 on about 3/10's of the trials, $30 on 6/10's, and $40 on 1/10. In calculating what average take per trial should be, you can assume 10 trials that follow the odds exactly:

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

3x20

3 hands will win $20 each for a total of $60

27

EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win $20 on about 3/10's of the trials, $30 on 6/10's, and $40 on 1/10. In calculating what average take per trial should be, you can assume 10 trials that follow the odds exactly:

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

3x20 + 6x30

6 hands will win $30 each for a total of $180

28

EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win $20 on about 3/10's of the trials, $30 on 6/10's, and $40 on 1/10. In calculating what average take per trial should be, you can assume 10 trials that follow the odds exactly:

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

3x20 + 6x30 + 1x40

1 hand will win $40 each for a total of $40

29

EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win $20 on about 3/10's of the trials, $30 on 6/10's, and $40 on 1/10. In calculating what average take per trial should be, you can assume 10 trials that follow the odds exactly:

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

3x20 + 6x30 + 1x40

TOTAL WINNINGS IN 10 REPRESENTATIVE TRIALS

30

EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win $20 on about 3/10's of the trials, $30 on 6/10's, and $40 on 1/10. In calculating what average take per trial should be, you can assume 10 trials that follow the odds exactly:

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

$$\frac{3 \times 20 + 6 \times 30 + 1 \times 40}{10} = 28$$

Average amount won per trial.

31

EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win $20 on about 3/10's of the trials, $30 on 6/10's, and $40 on 1/10.

Pr[(X=20)] = 3/10
Pr[(X=30)] = 6/10
Pr[(X=40)] = 1/10

$$\frac{3}{10} \times 20 + \frac{6}{10} \times 30 + \frac{1}{10} \times 40 = 28$$

$$\frac{3 \times 20 + 6 \times 30 + 1 \times 40}{10} = 28$$

Average amount won per trial.

32

EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win \$20 on about 3/10's of the trials, \$30 on 6/10's, and \$40 on 1/10.

$Pr[(X=20)] = 3/10$
$Pr[(X=30)] = 6/10$
$Pr[(X=40)] = 1/10$

$$\frac{3}{10}\times 20 + \frac{6}{10}\times 30 + \frac{1}{10}\times 40 = 28$$

$$\frac{3\times 20 + 6\times 30 + 1\times 40}{10} = 28$$

Average amount won per trial.

33

EXAMPLE: If you were to repeat this process repeatedly, and the 20, 30, and 40 represented your winnings in ($) on each trial, on average how much money would you make per trial?

EXAMPLE: If you ran the trial many, many times, you would expect to win \$20 on about 3/10's of the trials, \$30 on 6/10's, and \$40 on 1/10.

$Pr[(X=20)] = 3/10$
$Pr[(X=30)] = 6/10$
$Pr[(X=40)] = 1/10$

$$\frac{3}{10}\times 20 + \frac{6}{10}\times 30 + \frac{1}{10}\times 40 = 28$$

Average amount won per trial =

$$= Pr[(X=20)]\,20 + Pr[(X=30)]\,30 + Pr[(X=40)]\,40$$

34

DEFINITION: Let X be a random variable on a sample space that takes on values $x_1, x_2, \ldots x_p$. The expected value of X, called E(X), is given by:

$$E(X) = Pr[(X=x_1)]\,x_1 + Pr[(X=x_2)]\,x_2 + \cdots + Pr[(X=x_p)]\,x_p$$

Average amount won per trial =

$$Pr[(X=20)]\,20 + Pr[(X=30)]\,30 + Pr[(X=40)]\,40$$

35

DEFINITION: Let X be a random variable on a sample space that takes on values $x_1, x_2, \ldots x_p$. The expected value of X, called E(X), is given by:

$$E(X) = Pr[(X=x_1)]\,x_1 + Pr[(X=x_2)]\,x_2 + \cdots + Pr[(X=x_p)]\,x_p$$

Take each value that X takes on, then multiply it by the probability that it will occur. Add up all the numbers you get this way. That is E(X).

Average amount won per trial =

$$Pr[(X=20)]\,20 + Pr[(X=30)]\,30 + Pr[(X=40)]\,40$$

36

In practice, it helps to make a probability density table, listing the values of X and their probability of occurence;

Value of X	Probability
20	3/10
30	6/10
40	1/10

These add up to 1

37

In practice, it helps to make a probability density table, listing the values of X and their probability of occurence;

Value of X	Probability
20	3/10
30	6/10
40	1/10

These add up to 1

$E(X) = 20\ 3/10 + 30\ 6/10 + 40\ 1/10 = 28$

Standard probability + random variable → Probability density function & table → E(X)

38

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

39

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SOLUTION: Step 1: What values can X take on?

40

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION: Step 1: What values can X take on? 0, 1, 2, 3. The 3 tosses can result in 0, 1, 2, or 3 heads.

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION: Step 2: write down a "blank table," and then try to fill it in.

Value of X	Probability
0	___
1	___
2	___
3	___

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	___ Pr[(X=0)] =
1	___
2	___
3	___

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	___ Pr[(X=0)] = Pr[0H&3T] =
1	___
2	___
3	___

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	___ Pr[(X=0)] = Pr[0H&3T] = $(.4)^3 = .064$
1	___
2	___
3	___

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	___
2	___
3	___

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	___
2	___
3	___

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	___ Pr[(X=1)] =
2	___
3	___

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	___ Pr[(X=1)] = Pr[1H&2T] =
2	___
3	___

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	___ Pr[(X=1)] = Pr[1H&2T] = $C(3,1)(.6)^1(.4)^2$
2	___
3	___

$= 3\,.6(.4)^2 = .288$

50

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	.288
2	___
3	___

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	.288
2	___ Pr[(X=2)] =
3	___

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	.288
2	___ Pr[(X=2)] = Pr[2H&1T] =
3	___

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	.288
2	___ Pr[(X=2)] = Pr[2H&1T] = $C(3,2)(.6)^2(.4)^1$
3	___

$= 3\cdot(.6)^2(.4)^1 = .432$

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	.288
2	.432
3	___

55

EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	.288
2	.432
3	.216 ⟵ 1 - .784

.064 + .288 + .432 = .784

(ALL OF THE ENTRIES ADD UP TO ONE)

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability
0	.064
1	.288
2	.432
3	.216

This completely describes the probability density function.

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION: Step 3: Calculate E(X).

Value of X	Probability
0	.064
1	.288
2	.432
3	.216

This completely describes the probability density function.

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability	$x_i \cdot Pr(X=x_i)$
0	.064	
1	.288	
2	.432	
3	.216	

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

Value of X	Probability	$x_i \cdot Pr(X=x_i)$
0	.064	0
1	.288	.288
2	.432	.864
3	.216	.648

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

$$E(X) = 0(.064)+1(.288)+2(.432)+3(.216)$$
$$= 0(.064)+1(.288)+2(.432)+3(.216)$$
$$= 1.8$$

Value of X	Probability	$x_i \cdot Pr(X=x_i)$
0	.064	0
1	.288	.288
2	.432	.864
3	.216	.648
		1.8 = E(X)

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Find the probability density function for X and find E(X).

SOLUTION:

$$E(X) = 0(.064)+1(.288)+2(.432)+3(.216)$$
$$= 0(.064)+1(.288)+2(.432)+3(.216)$$
$$= 1.8$$

Value of X	Probability	$x_i \cdot Pr(X=x_i)$
0	.064	0
1	.288	.288
2	.432	.864
3	.216	.648
		1.8 = E(X)

EASY WAY: 3 x .6 = 1.8 This works in general for a binomial random variable: E(X) = np

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Y = (X - 2)^2$. Find the probability density function for Y and find E(Y).

Value of X	Probability
0	.064
1	.288
2	.432
3	.216

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Y = (X - 2)^2$. Find the probability density function for Y and find E(Y).

QUESTION: What values does Y take on?

Value of X	Probability
0	.064
1	.288
2	.432
3	.216

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Y = (X - 2)^2$. Find the probability density function for Y and find E(Y).

QUESTION: What values does Y take on?

Value of X	Probability	Value of Y	
0	.064	4	
1	.288	1	
2	.432	0	
3	.216	1	

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Y = (X - 2)^2$. Find the probability density function for Y and find E(Y).

Value of X	Probability	Value of Y	$y_i \cdot Pr(Y=y_i)$
0	.064	4	.256
1	.288	1	.288
2	.432	0	0
3	.216	1	.216

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Y = (X - 2)^2$. Find the probability density function for Y and find E(Y).

SOLUTION:

Value of X	Probability	Value of Y	$y_i \cdot Pr(Y=y_i)$
0	.064	4	.256
1	.288	1	.288
2	.432	0	0
3	.216	1	.216
			.770 = E(Y)

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Z = (X - 1.8)^2$. Find the probability density function for Z and find E(Z).

SOLUTION:

Value of X	Probability	Value of Z	$z_i \cdot Pr(Z=z_i)$
0	.064		
1	.288		
2	.432		
3	.216		

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Z = (X - 1.8)^2$. Find the probability density function for Z and find E(Z).

SOLUTION:

Value of X	Probability	Value of Z	$z_i \cdot Pr(Z=z_i)$
0	.064	3.24	
1	.288	.64	
2	.432	.04	
3	.216	1.44	

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Z = (X - 1.8)^2$. Find the probability density function for Z and find E(Z).

SOLUTION:

Value of X	Probability	Value of Z	$z_i \cdot Pr(Z=z_i)$
0	.064	3.24	.20736
1	.288	.64	.18432
2	.432	.04	.01728
3	.216	1.44	.31104

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Z = (X - 1.8)^2$. Find the probability density function for Z and find E(Z).

SOLUTION:

Value of X	Probability	Value of Z	$z_i \cdot Pr(Z=z_i)$
0	.064	3.24	.20736
1	.288	.64	.18432
2	.432	.04	.01728
3	.216	1.44	.31104
			.72 = E(Z)

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EXAMPLE: An unfair coin has probability of .6 of landing heads on any given toss. Let X = number of heads that result from flipping the coin 3 times. Let $Z = (X - 1.8)^2$. Find the probability density function for Z and find E(Z).

SOLUTION:

Value of X	Probability	Value of Z	$z_i \cdot Pr(Z=z_i)$
0	.064	3.24	.20736
1	.288	.64	.18432
2	.432	.04	.01728
3	.216	1.44	.31104
			.72 = E(Z)

Easy way: For a binomial random variable X:

$E[(X - E(X))^2] = npq$ where $q = 1 - p$.

E(Z) = 3(.6)(.4) = .72

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DEFINITION: Let X be a random variable on a sample space with $E(X) = \mu$. Let $Z = (X - \mu)^2$. Notice that Z is just another random variable. The variance of X is denoted by var(X) and is given by:

$$var(X) = E((X-\mu)^2) = E(Z)$$

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DEFINITION: Let X be a random variable on a sample space with $E(X) = \mu$. Let $Z = (X - \mu)^2$. Notice that Z is just another random variable. The variance of X is denoted by var(X) and is given by:

$$var(X) = E((X-\mu)^2) = E(Z)$$

Furthermore, the standard deviation of X is denoted by $\sigma(X)$ and is defined as:

$$\sigma(X) \equiv \sqrt{var(X)}$$

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THEOREM: If X is a binomial random variable for a Bernoulli process with n trials and a probability p of success on each trial, then

$$E(X) = np.$$

Furthermore, the variance and standard deviation of X are given by:

$$var(X) = npq \qquad \sigma(X) = \sqrt{npq}$$

Note: $q = 1 - p$

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THEOREM: If X is a binomial random variable for a Bernoulli process with n trials and a probability p of success on each trial, then

$$E(X) = np.$$

Furthermore, the variance and standard deviation of X are given by:

$$var(X) = npq \qquad \sigma(X) = \sqrt{npq}$$

CAUTION: THESE FORMULAS DON'T WORK FOR MOST RANDOM VARIABLES

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