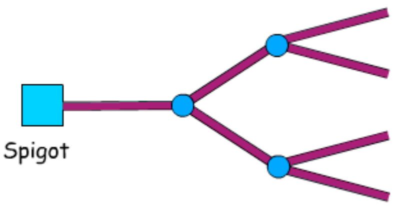
CONDITIONAL PROBABILITY AND TREES:

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



1

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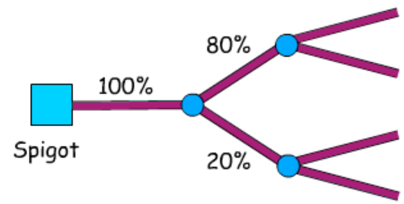
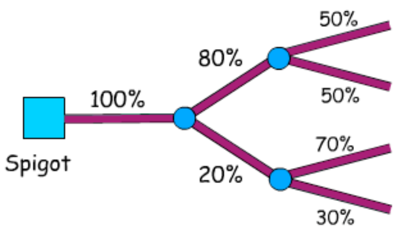


Make sure that you have adjusted the first junction box so that 80% of the water goes to the "upper" line and 20% to the lower.

2

CONDITIONAL PROBABILITY AND TREES:

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:

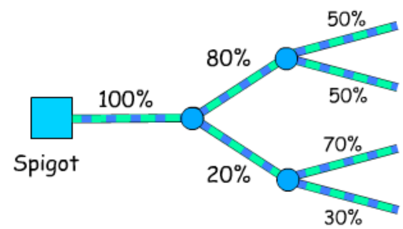


Adjust the other junction boxes as shown. And, turn on the water.

3

CONDITIONAL PROBABILITY AND TREES:

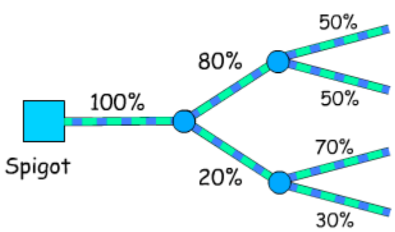
HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



4

CONDITIONAL PROBABILITY AND TREES:

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:

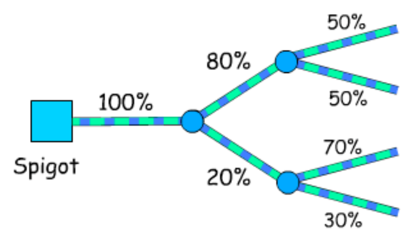


Question: How much water comes out right here: ←

5

CONDITIONAL PROBABILITY AND TREES:

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



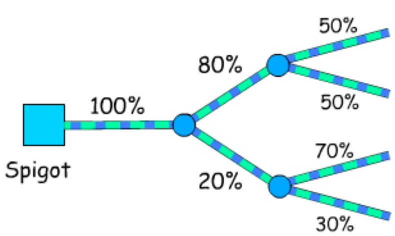
Answer: ←

70% of 20%

6

CONDITIONAL PROBABILITY AND TREES:

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



Answer: ←

70% of 20%

.7 x .2 = .14

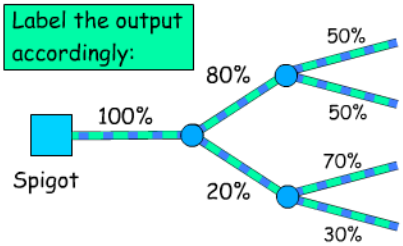
14%

7

CONDITIONAL PROBABILITY AND TREES:

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:

Label the output accordingly:



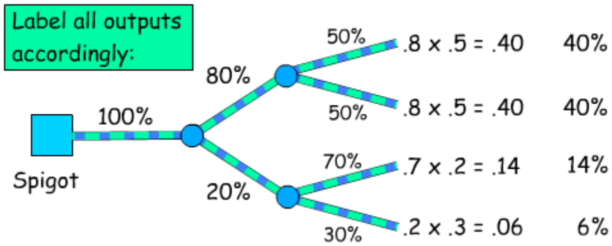
.7 x .2 = .14 14%

8

CONDITIONAL PROBABILITY AND TREES:

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:

Label all outputs accordingly:

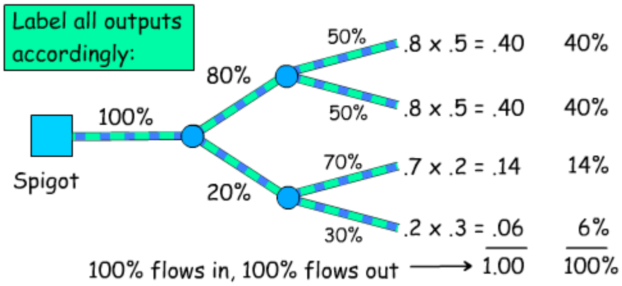


9

CONDITIONAL PROBABILITY AND TREES:

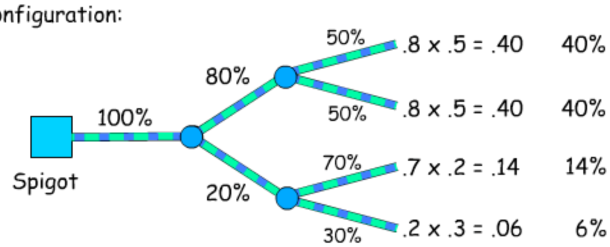
HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:

Label all outputs accordingly:



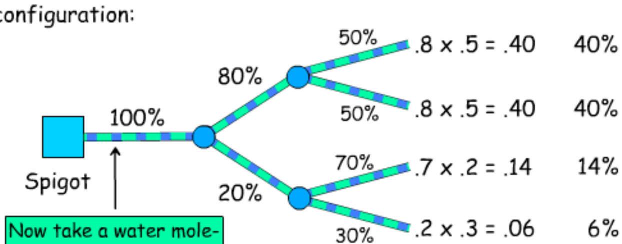
10

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



11

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:

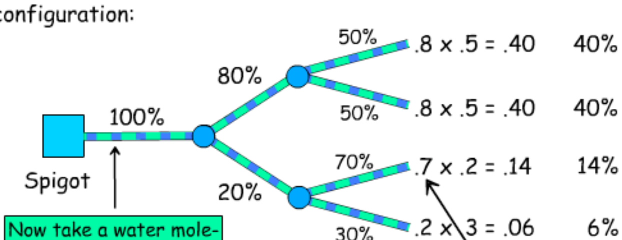


Now take a water molecule and paint it red. Release it right here.

Assume the water is extremely turbulent.

12

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



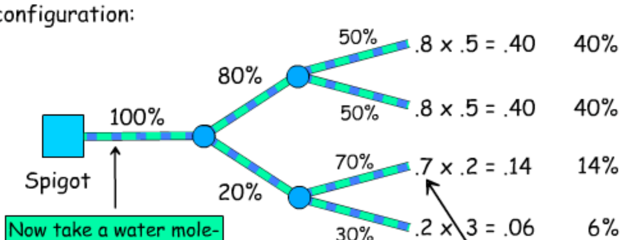
Now take a water molecule and paint it red. Release it right here.

Assume the water is extremely turbulent.

QUESTION: What is the probability that it came out right here?

13

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



Now take a water molecule and paint it red. Release it right here.

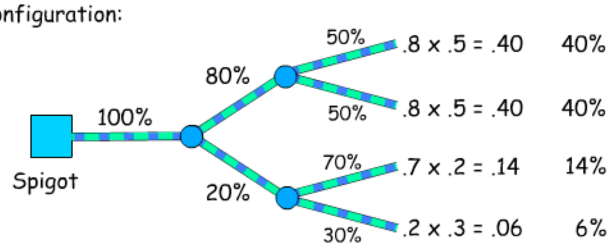
Assume the water is extremely turbulent.

QUESTION: What is the probability that it came out right here?

ANSWER: .14 (14% of H_2O exits here)

14

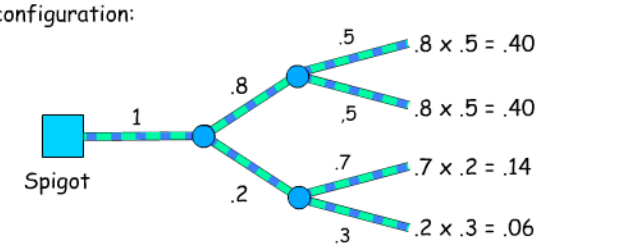
HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:



Percentages translate directly into probabilities:

15

HOME PROJECT: Go out to your front yard and cut up your garden hose, and tape it and glue it together in the following configuration:

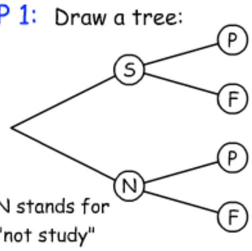


If the route of a particle is determined probabilistically as indicated above, then the numbers at the right indicate the probabilities for the outcome at the given locations.

16

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing?

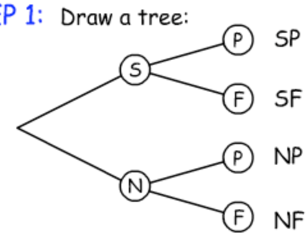
STEP 1: Draw a tree:



N stands for "not study"

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing?

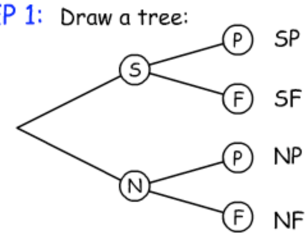
STEP 1: Draw a tree:



Officially, these are the outcomes. We'll never mention them again using this notation.

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing?

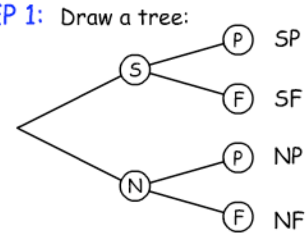
STEP 1: Draw a tree:



Officially, these are the outcomes. We'll never mention them again using this notation. However, S will be used to denote {SP,SF} an event in the sample space (i.e anything with an S in it).

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing?

STEP 1: Draw a tree:



Likewise, F will denote {SF, NF}. It represents the event that he failed. So the notation makes sense.

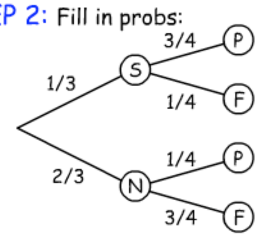
There is also the event N, the event that he does not study.

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

The problem asks, what is Pr[P]?

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

STEP 2: Fill in probs:



EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

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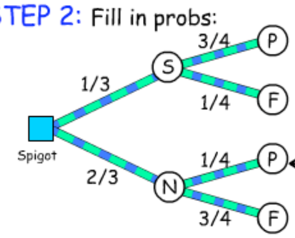

At this point you could build a garden hose model of this. A red molecule released at the spigot would follow these probs. The probability that it would come out here would be the same as the probability that Clyde would not study and would pass.

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

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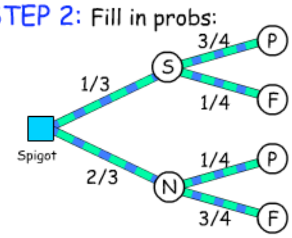
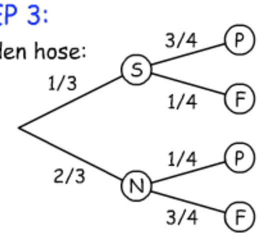


Clyde's life is pretty much modeled by this red water molecule. There is a 1/3 chance the water molecule will reach S, and there is a 1/3 chance Clyde will study. If the molecule makes it to S, there is a 3/4 it will make it to P, and if Clyde studies there is a 3/4 he will pass. The garden hose model should work.

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

STEP 3:

Garden hose:



1/3 x 3/4 = 1/4

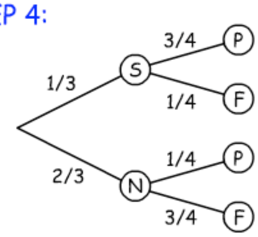
1/3 x 1/4 = 1/12

2/3 x 1/4 = 1/6

2/3 x 3/4 = 1/2

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

STEP 4:



1/3 x 3/4 = 1/4 ←

1/3 x 1/4 = 1/12

2/3 x 1/4 = 1/6 ←

2/3 x 3/4 = 1/2

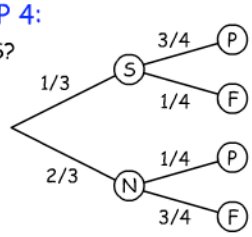
Passes here and here.

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

STEP 4:

PASS?

Pr[P]



1/3 x 3/4 = 1/4 ←

1/3 x 1/4 = 1/12

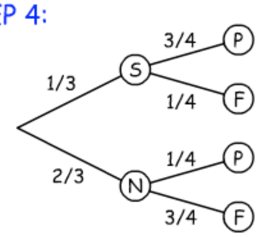
2/3 x 1/4 = 1/6 ←

2/3 x 3/4 = 1/2

1/4 + 1/6 = how much water flows out here

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?

STEP 4:



1/3 x 3/4 = 1/4 ←

1/3 x 1/4 = 1/12

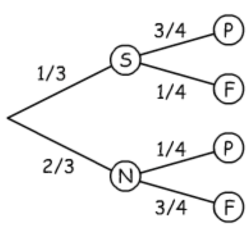
2/3 x 1/4 = 1/6 ←

2/3 x 3/4 = 1/2

1/4 + 1/6 = Pr[P]

Pr[Pass] = 5/12

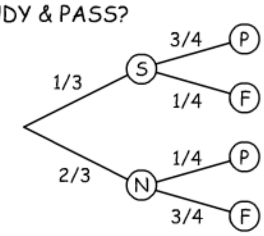
EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? Pr[P] = ?



	P	
1/3 x 3/4 = 1/4	X	
1/3 x 1/4 = 1/12		
2/3 x 1/4 = 1/6	X	
2/3 x 3/4 = 1/2		
	5/12	

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? PR[S∩P] = ?

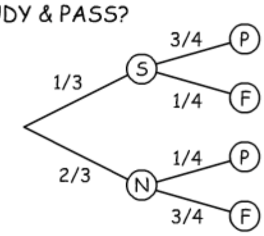
STUDY & PASS?



	P	S	S∩P
1/3 x 3/4 = 1/4	X		
1/3 x 1/4 = 1/12			
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? PR[S∩P] = ?

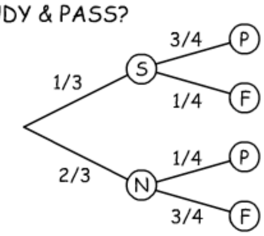
STUDY & PASS?



	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		

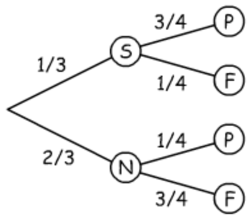
EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is his probability of passing? PR[S∩P] = ?

STUDY & PASS?



	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	X
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		1/4

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed?



	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	X
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		1/4

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed?

COMMENT: This is a conditional probability problem. You are being given addtional information.

Given that he passed, what is the probability that he studied?

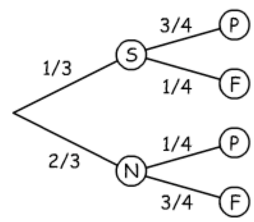
EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed?

COMMENT: This is a conditional probability problem. You are being given addtional information.

Given that he passed, what is the probability that he studied?

Pr[S|P] = ?

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?



	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	X
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		1/4

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

$$\Pr[S|P] = \frac{\Pr[S \cap P]}{\Pr[P]}$$

	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	X
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		1/4

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

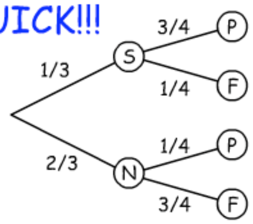
$$\Pr[S|P] = \frac{\Pr[S \cap P]}{\Pr[P]} =$$

$$= \frac{1/4}{5/12} = 3/5$$

	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	X
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		1/4

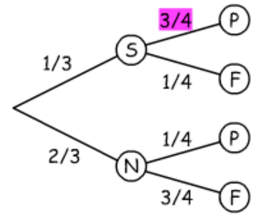
EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he passed IF he studied? Pr[P|S] = ?

QUICK!!!



	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	X
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		1/4

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he passed IF he studied? Pr[P|S] = 3/4



	P	S	S∩P
1/3 x 3/4 = 1/4	X	X	X
1/3 x 1/4 = 1/12		X	
2/3 x 1/4 = 1/6	X		
2/3 x 3/4 = 1/2			
	5/12		1/4

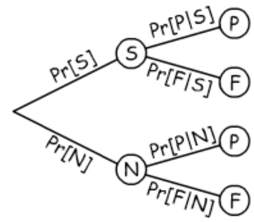
EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? GO BACK TO EARLIER PROBLEM

Return to: Pr[S|P] = ?

Now we'll look at the problem using symbols rather than numbers.

41

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?



	P	S	S∩P
Pr[S} Pr[P|S]	X	X	X
Pr[S} Pr[F|S]		X	
Pr[N} Pr[P|N]	X		
Pr[N} Pr[F|N]			

42

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

$$Pr[S|P] = \frac{Pr[S \cap P]}{Pr[P]} =$$

	P	S	S∩P
Pr[S} Pr[P|S]	X	X	X
Pr[S} Pr[F|S]		X	
Pr[N} Pr[P|N]	X		
Pr[N} Pr[F|N]			

43

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

$$Pr[S|P] = \frac{Pr[S \cap P]}{Pr[P]} =$$

$$= \frac{Pr[S] \cdot Pr[P|S]}{Pr[S]\, Pr[P|S] + Pr[N]\, Pr[P|N]}$$

	P	S	S∩P
Pr[S} Pr[P|S]	X	X	X
Pr[S} Pr[F|S]		X	
Pr[N} Pr[P|N]	X		
Pr[N} Pr[F|N]			

44

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

$$Pr[S|P] = \frac{Pr[S \cap P]}{Pr[P]} =$$

BAYE'S FORMULA

prob of passing by studying

$$= \frac{Pr[S] \cdot Pr[P|S]}{Pr[S]\, Pr[P|S] + Pr[N]\, Pr[P|N]}$$

On the dart board of passing, how large is studying?

prob of passing by studying or not studying (either way)

45

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

Pr[S] = 1/3 Pr[N] = 2/3

Pr[P|S] = 3/4 Pr[P|N] = 1/4

Pr[S|P] = ?

$$Pr[S|P] = \frac{Pr[S \cap P]}{Pr[P]} =$$

$$= \frac{Pr[S] \cdot Pr[P|S]}{Pr[S]\, Pr[P|S] + Pr[N]\, Pr[P|N]}$$

46

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

Pr[S] = 1/3 Pr[N] = 2/3

Pr[P|S] = 3/4 Pr[P|N] = 1/4

Pr[S|P] = ?

$$Pr[S|P] = \frac{Pr[S \cap P]}{Pr[P]} =$$

1/3 · 3/4

$$= \frac{Pr[S] \cdot Pr[P|S]}{Pr[S]\, Pr[P|S] + Pr[N]\, Pr[P|N]}$$

1/3 · 3/4 + 2/3 · 1/4

47

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? Pr[S|P] = ?

Pr[S] = 1/3 Pr[N] = 2/3

Pr[P|S] = 3/4 Pr[P|N] = 1/4

Pr[S|P] = ?

$$Pr[S|P] = \frac{Pr[S \cap P]}{Pr[P]} =$$

$$= \frac{1/3 \cdot 3/4}{1/3 \cdot 3/4 + 2/3 \cdot 1/4}$$

48

EXAMPLE: Clyde studies for 1/3 of his exams (chosen at random). If he studies, he has a probability 3/4 of passing an exam. If he doesn't study, he has a probability of 1/4 of passing. On a randomly selected test, what is probability that he studied IF he passed? $Pr[S|P]$ = ?

$Pr[S] = 1/3$ $Pr[N] = 2/3$

$Pr[P|S] = 3/4$ $Pr[P|N] = 1/4$

$Pr[S|P]$ = ?

$$Pr[S|P] = \frac{Pr[S \cap P]}{Pr[P]} = \frac{1/3 \cdot 3/4}{1/3 \cdot 3/4 + 2/3 \cdot 1/4}$$

$= 3/5$

49

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

50

Lecture 11

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $Pr[W_1|W_2]$

SOLUTION:

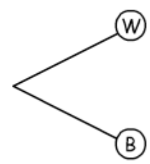
W_1 is the event of getting a white on the first draw.

W_2 is the event of getting a white on the second draw.

51

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $Pr[W_1|W_2]$

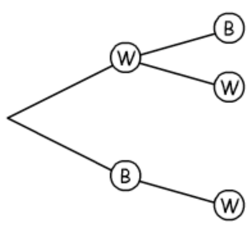
SOLUTION:



52

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $Pr[W_1|W_2]$

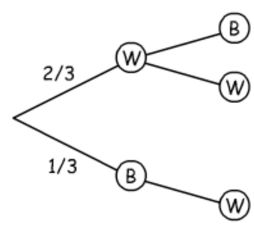
SOLUTION:



53

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $Pr[W_1|W_2]$

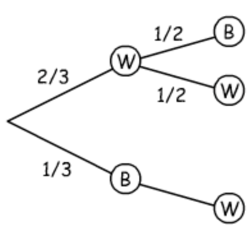
SOLUTION:



54

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $Pr[W_1|W_2]$

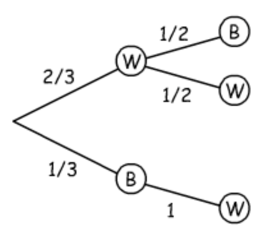
SOLUTION:



55

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $Pr[W_1|W_2]$

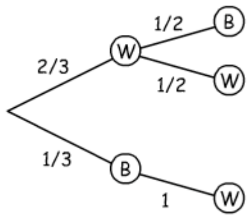
SOLUTION:



56

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $\Pr[W_1|W_2]$

SOLUTION:

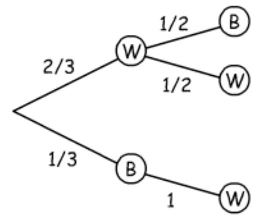


2/3 x 1/2 = 1/3			
2/3 x 1/2 = 1/3			
1/3 x 1 = 1/3			

57

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $\Pr[W_1|W_2]$

SOLUTION:



	W_1	W_2	
2/3 x 1/2 = 1/3	X		
2/3 x 1/2 = 1/3	X	X	
1/3 x 1 = 1/3		X	

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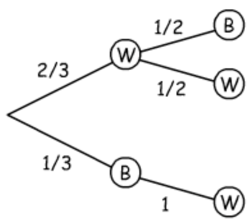
Lecture 11

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $\Pr[W_1|W_2]$

SOLUTION:

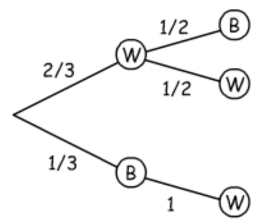


	W_1	W_2	
2/3 x 1/2 = 1/3	X		
2/3 x 1/2 = 1/3	X	X	
1/3 x 1 = 1/3		X	
	2/3	2/3	

59

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $\Pr[W_1|W_2]$

SOLUTION:



	W_1	W_2	$W_1 \cap W_2$
2/3 x 1/2 = 1/3	X		
2/3 x 1/2 = 1/3	X	X	X
1/3 x 1 = 1/3		X	
	2/3	2/3	1/3

60

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white? $\Pr[W_1|W_2]$

SOLUTION:

$$\Pr[W_1|W_2] = \frac{\Pr[W_1 \cap W_2]}{\Pr[W_2]} =$$

$$= \frac{1/3}{2/3} = 1/2$$

	W_1	W_2	$W_1 \cap W_2$
2/3 x 1/2 = 1/3	X		
2/3 x 1/2 = 1/3	X	X	X
1/3 x 1 = 1/3		X	
	2/3	2/3	1/3

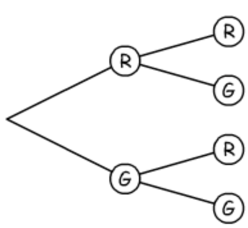
61

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

62

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

TREE:



63

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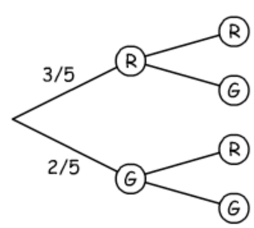
TREE:

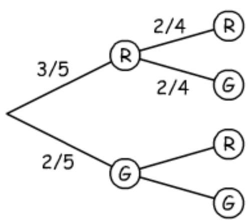


64

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

TREE:



65

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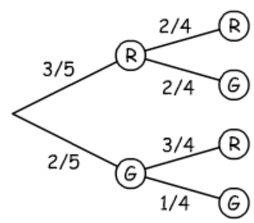
ERASE

TREE:



66

Lecture 11

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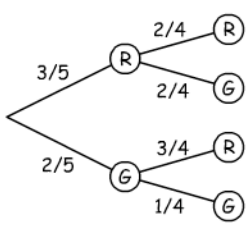
ERASE

TREE:



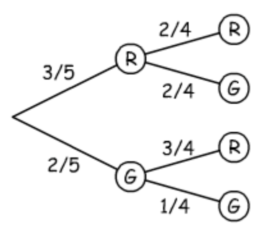
3/5 x 2/4 = 3/10			
3/5 x 2/4 = 3/10			
2/5 x 3/4 = 3/10			
2/5 x 1/4 = 1/10			

67

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

TREE:



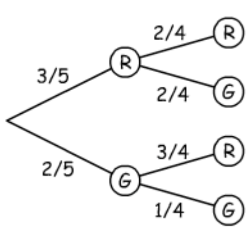
	2R	A1R	2R ∩ A1R
3/5 x 2/4 = 3/10	X		
3/5 x 2/4 = 3/10			
2/5 x 3/4 = 3/10			
2/5 x 1/4 = 1/10			

68

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

TREE:



	2R	A1R	2R ∩ A1R
3/5 x 2/4 = 3/10	X	X	
3/5 x 2/4 = 3/10		X	
2/5 x 3/4 = 3/10		X	
2/5 x 1/4 = 1/10			

69

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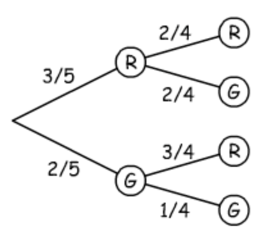
ERASE

TREE:



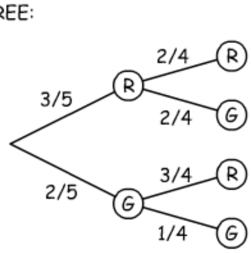
	2R	A1R	2R ∩ A1R
3/5 x 2/4 = 3/10	X	X	X
3/5 x 2/4 = 3/10		X	
2/5 x 3/4 = 3/10		X	
2/5 x 1/4 = 1/10			

70

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

TREE:



	2R	A1R	2R ∩ A1R
3/5 x 2/4 = 3/10	X	X	X
3/5 x 2/4 = 3/10		X	
2/5 x 3/4 = 3/10		X	
2/5 x 1/4 = 1/10			
		9/10	3/10

71

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]}$$

$$= \frac{3/10}{9/10} = 1/3$$

	2R	A1R	2R ∩ A1R
3/5 x 2/4 = 3/10	X	X	X
3/5 x 2/4 = 3/10		X	
2/5 x 3/4 = 3/10		X	
2/5 x 1/4 = 1/10			
		9/10	3/10

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