CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

This involves calculating probabilities given some sort of extra information.

1

CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

EXAMPLE: 1) A hat contains 3 slips of paper, 2 white and 1 black. You remove, at random, one slip of paper, but DON'T look at it. What is the probability that you got a white slip?

2

CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

EXAMPLE: 1) A hat contains 3 slips of paper, 2 white and 1 black. You remove, at random, one slip of paper, but DON'T look at it. What is the probability that you got a white slip?

Answer: Pr[You drew a white] = 2/3

3

CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

EXAMPLE: 1) A hat contains 3 slips of paper, 2 white and 1 black. You remove, at random, one slip of paper, but DON'T look at it. What is the probability that you got a white slip?

Answer: Pr[You drew a white] = 2/3

2) Next, someone draws a slip from the hat at random from the two remaining slips, and it is white. Now, what is the probability that you got a white slip?

4

CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

EXAMPLE: 1) A hat contains 3 slips of paper, 2 white and 1 black. You remove, at random, one slip of paper, but DON'T look at it. What is the probability that you got a white slip?

Answer: Pr[You drew a white] = 2/3

2) Next, someone draws a slip from the hat at random from the two remaining slips, and it is white. Now, what is the probability that you got a white slip?

Answer: Your situation is the same as if, in the beginning, a white slip was drawn out (rather than after your draw). So, effectively, from a white slip and a black slip, you select one at random. Pr[You drew a white] = 1/2

5

CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

EXAMPLE: 1) A hat contains 3 slips of paper, 2 white and 1 black. You remove, at random, one slip of paper, but DON'T look at it. What is the probability that you got a white slip?

Answer: Pr[You drew a white] = 2/3

2) Next, someone draws a slip from the hat at random from the two remaining slips, and it is white. Now, what is the probability that you got a white slip?

What is the difference between 1) and 2)?

Pr[You drew a white] = 1/2

6

CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

EXAMPLE: 1) A hat contains 3 slips of paper, 2 white and 1 black. You remove, at random, one slip of paper, but DON'T look at it. What is the probability that you got a white slip?

Answer: Pr[You drew a white] = 2/3

2) Next, someone draws a slip from the hat at random from the two remaining slips, and it is white. Now, what is the probability that you got a white slip?

Answer: You are given additional information. You are told the results of another draw.

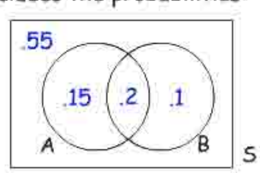
Pr[You drew a white] = 1/2

7

CONDITIONAL PROBABILITY: Venn diagram approach

Let A and B be subsets of a sample space S. Here's a Venn diagram which includes the probabilities:

This is just an example. Obviously, the actual numbers may vary.

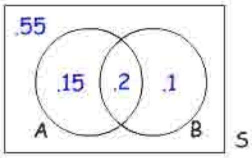


8

CONDITIONAL PROBABILITY: Venn diagram approach

Let A and B be subsets of a sample space S. Here's a Venn diagram which includes the probabilities:

This is just an example. Obviously, the actual numbers may vary.



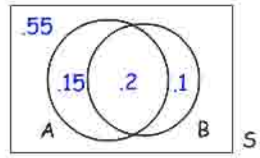
You could, if you were careful, draw the diagram so that S has area 1 (in what ever units you are using), and the sets have each have an area equal to the probabilities shown in the diagram.

9

CONDITIONAL PROBABILITY: Venn diagram approach

Let A and B be subsets of a sample space S. Here's a Venn diagram which includes the probabilities:

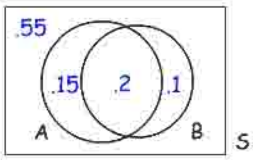
Now drawn to scale.



10

CONDITIONAL PROBABILITY: Venn diagram approach

Let A and B be subsets of a sample space S. Here's a Venn diagram which includes the probabilities:

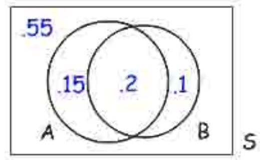


Now pretend that you have magic darts, such that when thrown always hit S and land randomly on S. Then the probability of hitting A on any one toss is .35, and the probability of hitting B is .3, and the probability of hitting $A \cap B$ is .2. The probability of missing A and B is .55.

11

CONDITIONAL PROBABILITY: Venn diagram approach

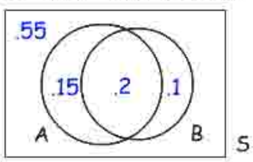
Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



12

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?

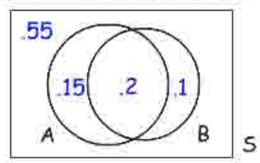


The dart is assumed to be one of the magic darts, that it is just as likely to hit one point in S as another. However, in this case, a reliable witness tells you that it hit B. This is all you know other than it landed "at random."

13

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?

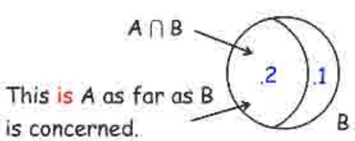


Answer: You are just as likely to hit any given point in B as any other point. It is as if B has become the entire sample space (i.e. THE ENTIRE DART BOARD), and the dart lands randomly on this new board.

14

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?

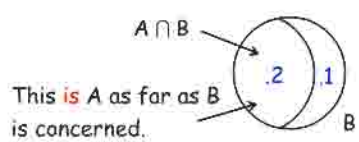


Answer: You are just as likely to hit any given point in B as any other point. It is as if B has become the entire sample space (i.e. THE ENTIRE DART BOARD), and the dart lands randomly on this new board.

15

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



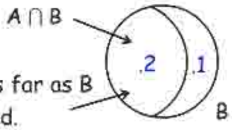
The entire new "dart board" has area .3. The set $A \cap B$ has area .2.

So A accounts for $\frac{.2}{.3} = \frac{2}{3}$ of this new dart board.

16

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



This is A as far as B is concerned.

The entire new "dart board" has area .3. The set $A \cap B$ has area .2.

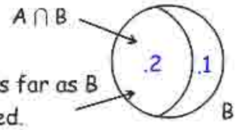
So A accounts for $\frac{.2}{.3} = \frac{2}{3}$ of this new dart baord.

If a dart hits at a random place on B, then the probability that it hits A is 2/3.

17

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



This is A as far as B is concerned.

The entire new "dart board" has area .3. The set $A \cap B$ has area .2.

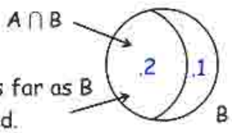
So A accounts for $\frac{.2}{.3} = \frac{2}{3}$ of this new dart baord.

If a dart hits at a random place on B, then the probability that it hits A is 2/3. ANSWER: 2/3

18

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



This is A as far as B is concerned.

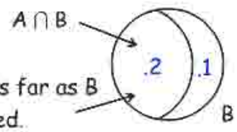
Notice that the answer 2/3 is

$$\frac{\text{area}(A \cap B)}{\text{area}(B)} = \frac{\Pr(A \cap B)}{\Pr(B)}$$

19

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



This is A as far as B is concerned.

Notice that the answer 2/3 is

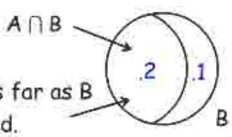
$$\frac{\text{area}(A \cap B)}{\text{area}(B)} = \frac{\Pr(A \cap B)}{\Pr(B)} = \Pr[A|B]$$

The official language for the probability of hitting A knowing that you hit B is $\Pr[A|B]$.

20

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



This is A as far as B is concerned.

Notice that the answer 2/3 is

$$\frac{\text{area}(A \cap B)}{\text{area}(B)} = \frac{\Pr(A \cap B)}{\Pr(B)} = \Pr[A|B]$$

The official language for the probability of hitting A knowing that you hit B is $\Pr[A|B]$. We now have:

21

CONDITIONAL PROBABILITY:

Definition: Let A and B be subsets of a sample space S. Provided $\Pr[B] \neq 0$, then the probability that A occurs given that B has occured is defined as:

$$\Pr[A|B] = \frac{\Pr(A \cap B)}{\Pr(B)}$$

22

CONDITIONAL PROBABILITY:

Definition: Let A and B be subsets of a sample space S. Provided $\Pr[B] \neq 0$, then the probability that A occurs given that B has occured is defined as:

$$\Pr[A|B] = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Effectively this is

$$\Pr[A|B] = \frac{\text{area}(A \cap B)}{\text{area}(B)}.$$

23

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

24

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white.

25

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white.

Usually, you don't even bother to write out the sample space, but in this case here it is: (slips W, W*, and B are in the hat)

$$S = \{(W,W^*),(W^*,W),(W,B),(B,W),(W^*,B),(B,W^*)\}$$

$n(S)=6$ $n(W_1) = 4$ $n(W_1 \cap W_2) = n(\{(W,W^*),(W^*,W)\}) = 2$

26

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = ?$$

27

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = \frac{\Pr(W_1 \cap W_2)}{\Pr(W_2)}$$

28

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = \frac{\Pr(W_1 \cap W_2)}{\Pr(W_2)}$$

Finding the numerator is equivalent to solving:

If 2 slips are drawn, one after another, without replacement what is the probability of drawing a white and then a white.

29

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = \frac{\Pr(W_1 \cap W_2)}{\Pr(W_2)}$$

$$\Pr(W_1 \cap W_2) = \Pr[\text{drawing 2 whites}] = \frac{\text{\# ways to draw 2 W}}{\text{\# ways to draw any 2}}$$

30

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = \frac{\Pr(W_1 \cap W_2)}{\Pr(W_2)}$$

$$\Pr(W_1 \cap W_2) = \Pr[\text{drawing 2 whites}] = \frac{\text{\# ways to draw 2 W}}{\text{\# ways to draw any 2}}$$

$$= 1/3$$

31

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = \frac{\Pr(W_1 \cap W_2)}{\Pr(W_2)} = \frac{1/3}{}$$

32

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$Pr[W_1|W_2] = \frac{Pr(W_1 \cap W_2)}{Pr(W_2)} = \frac{1/3}{\quad}$$

Denominator: $Pr(W_2)$

33

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$Pr[W_1|W_2] = \frac{Pr(W_1 \cap W_2)}{Pr(W_2)} = \frac{1/3}{\quad}$$

Denominator: $Pr(W_2) = 2/3$

34

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$Pr[W_1|W_2] = \frac{Pr(W_1 \cap W_2)}{Pr(W_2)} = \frac{1/3}{\quad}$$

2/3

35

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$Pr[W_1|W_2] = \frac{Pr(W_1 \cap W_2)}{Pr(W_2)} = \frac{1/3}{2/3}$$

36

EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$Pr[W_1|W_2] = \frac{Pr(W_1 \cap W_2)}{Pr(W_2)} = \frac{1/3}{2/3}$$

$$= \frac{1}{2}$$

37

EXAMPLE: Suppose E and F are events in a sample space S with $Pr[E] = .65$, $Pr[F] = .4$ and $Pr[E \cup F] = .75$. Find

a) $Pr[E|F]$

b) $Pr[E|F^c]$

38

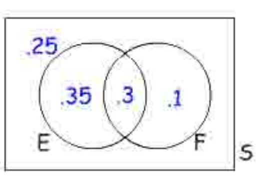
EXAMPLE: Suppose E and F are events in a sample space S with $Pr[E] = .65$, $Pr[F] = .4$ and $Pr[E \cup F] = .75$. Find

a) $Pr[E|F]$

b) $Pr[E|F^c]$

SOLUTION: Earlier, the Venn diagram for this set up was found.

Venn Diagram:



39

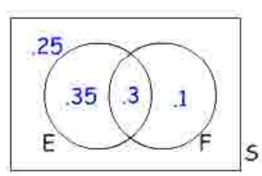
EXAMPLE: Suppose E and F are events in a sample space S with $Pr[E] = .65$, $Pr[F] = .4$ and $Pr[E \cup F] = .75$. Find

a) $Pr[E|F]$

b) $Pr[E|F^c]$

SOLUTION:

$$Pr[E|F] = \frac{Pr[E \cap F]}{Pr[F]}$$



40

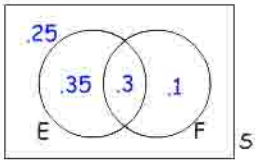
EXAMPLE: Suppose E and F are events in a sample space S with Pr[E] = .65, Pr[F] = .4 and Pr[E ∪ F] = .75. Find

a) Pr[E|F]

b) $Pr[E|F^c]$

SOLUTION:

$$Pr[E|F] = \frac{Pr[E \cap F]}{Pr[F]} = \frac{.3}{.4} = \frac{3}{4}$$



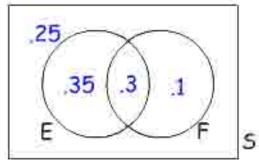
EXAMPLE: Suppose E and F are events in a sample space S with Pr[E] = .65, Pr[F] = .4 and Pr[E ∪ F] = .75. Find

a) Pr[E|F]

b) $Pr[E|F^c]$

SOLUTION:

$$Pr[E|F^c] =$$



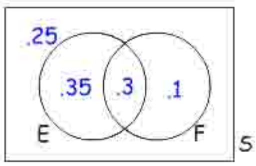
EXAMPLE: Suppose E and F are events in a sample space S with Pr[E] = .65, Pr[F] = .4 and Pr[E ∪ F] = .75. Find

a) Pr[E|F]

b) $Pr[E|F^c]$

SOLUTION:

$$Pr[E|F^c] = \frac{Pr[E \cap F^c]}{Pr[F^c]}$$



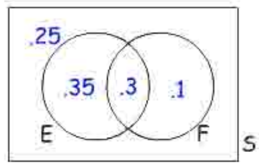
EXAMPLE: Suppose E and F are events in a sample space S with Pr[E] = .65, Pr[F] = .4 and Pr[E ∪ F] = .75. Find

a) Pr[E|F]

b) $Pr[E|F^c]$

SOLUTION:

$$Pr[E|F^c] = \frac{Pr[E \cap F^c]}{Pr[F^c]} = \frac{.35}{.6} = \frac{7}{12}$$



EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian

2R = event of getting two red martians

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian

2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]}$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian

2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]}$$

Pr[2R ∩ A1R] = ? 2R ∩ A1R = ?

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]}$$

$Pr[2R \cap A1R] = ?$ $\quad 2R \cap A1R = 2R$

49

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]} = \frac{Pr[2R]}{Pr[A1R]}$$

$Pr[2R \cap A1R] = ?$ $\quad 2R \cap A1R = 2R$

50

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]} = \frac{Pr[2R]}{Pr[A1R]}$$

$Pr[2R] =$

51

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]} = \frac{Pr[2R]}{Pr[A1R]}$$

$$Pr[2R] = \frac{\text{\# ways to select 2 reds}}{\text{\# ways to select any 2}}$$

52

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]} = \frac{Pr[2R]}{Pr[A1R]}$$

$$Pr[2R] = \frac{\text{\# ways to select 2 reds}}{\text{\# ways to select any 2}} = \frac{C(3,2)}{C(5,2)}$$

53

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]} = \frac{Pr[2R]}{Pr[A1R]}$$

$$Pr[2R] = \frac{\text{\# ways to select 2 reds}}{\text{\# ways to select any 2}} = \frac{C(3,2)}{C(5,2)} = 3/10$$

54

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]} = \frac{Pr[2R]}{Pr[A1R]} = \frac{3/10}{}$$

$$Pr[2R] = \frac{\text{\# ways to select 2 reds}}{\text{\# ways to select any 2}} = \frac{C(3,2)}{C(5,2)} =$$

55

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$Pr[2R|A1R] = \frac{Pr[2R \cap A1R]}{Pr[A1R]} = \frac{Pr[2R]}{Pr[A1R]} = \frac{3/10}{}$$

$Pr[A1R] =$

56

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{}$$

$\Pr[A1R] = 1 - \Pr[NR]$ (select no red; complementary events)

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{}$$

$$\Pr[A1R] = 1 - \Pr[NR] = 1 - \frac{\text{\# ways 2 green}}{\text{\# ways to select any 2}}$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{}$$

$$\Pr[A1R] = 1 - \Pr[NR] = 1 - \frac{\text{\# ways 2 green}}{\text{\# ways to select any 2}} = 1 - \frac{1}{10} = 9/10$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{9/10}$$

$$\Pr[A1R] = 1 - \Pr[NR] = 1 - \frac{\text{\# ways 2 green}}{\text{\# ways to select any 2}} = 1 - \frac{1}{10} =$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{9/10} = 1/3$$

CONDITIONAL PROBABILITY AND INDEPENDENCE:

INDEPENDENCE:

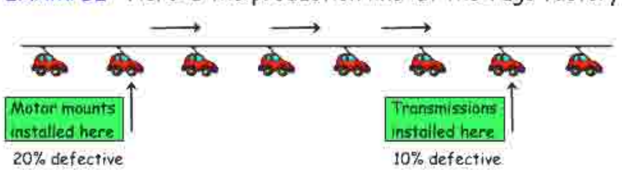
This is closely related to conditional probability.

CONDITIONAL PROBABILITY AND INDEPENDENCE:

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



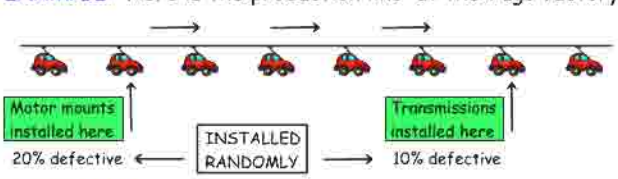
These parts come from the supplier and the defects are NOT visible. So the workers don't know they are installing a defective part or not. Whether or not a car gets a defective transmission has nothing to do with whether or not it got defective motor mounts.

CONDITIONAL PROBABILITY AND INDEPENDENCE:

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



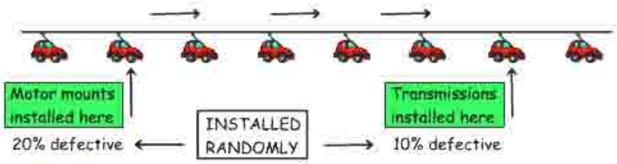
QUESTION: What % of the cars have a defective transmission and defective motor mounts.

CONDITIONAL PROBABILITY AND INDEPENDENCE:

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20%

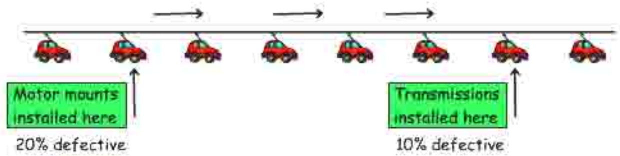
65

CONDITIONAL PROBABILITY AND INDEPENDENCE:

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



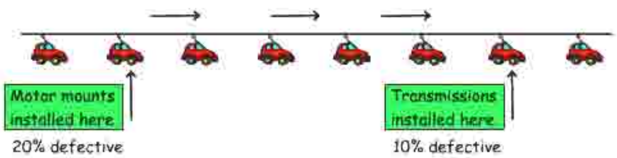
QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $\quad .1 \times .2 = .02 \Rightarrow 2\%$

66

Lecture 10

EXAMPLE: Here is the production line at the Yugo factory:

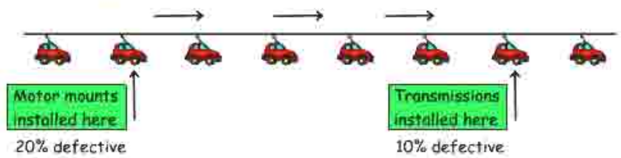


QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $\quad .1 \times .2 = .02 \Rightarrow 2\%$

67

EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $\quad .1 \times .2 = .02 \Rightarrow 2\%$

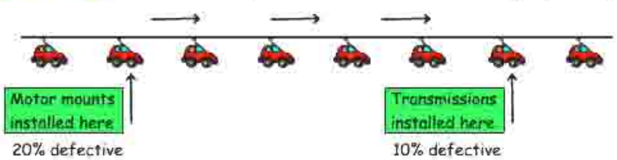
RESTATED: If a car is selected at random, let

DM = event of selecting a car with defective motor mounts

DT = event of selecting a car with a defective tranmission

68

EXAMPLE: Here is the production line at the Yugo factory:



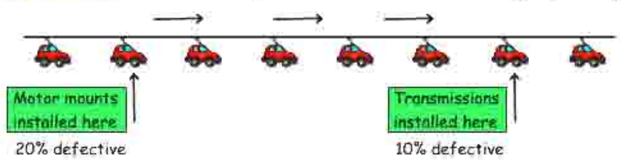
QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $\quad .1 \times .2 = .02 \Rightarrow 2\%$

RESTATED: $Pr[DM] = .2$ and $Pr[DT] = .1$

69

EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $\quad .1 \times .2 = .02 \Rightarrow 2\%$

RESTATED: $Pr[DM] = .2$ and $Pr[DT] = .1$ and

$$Pr[DM \cap DT] = Pr[DM] \cdot Pr[DT] = .1 \times .2 = .02$$

event of a defective trans. and motor mounts

70

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

This happens any time the two sets have "nothing to do with one another" as in the previous example.

71

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

This happens any time the two sets have "nothing to do with one another" as in the previous example.

72

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

NOTE: If A and B are independent with $Pr[B] \neq 0$, then

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]}$$

73

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

NOTE: If A and B are independent with $Pr[B] \neq 0$, then

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{Pr[A] \cdot Pr[B]}{Pr[B]} = Pr[A]$$

74

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

NOTE: If A and B are independent with $Pr[B] \neq 0$, then

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{Pr[A] \cdot Pr[B]}{Pr[B]} = Pr[A]$$

So the probability that A will occur given that B will occur is exactly the probability that A will occur. The fact that B has occured has no effect on the probability of A occuring.

75

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

NOTE: If A and B are independent with $Pr[B] \neq 0$, then

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{Pr[A] \cdot Pr[B]}{Pr[B]} = Pr[A]$$

So the probability that A will occur given that B will occur is exactly the probability that A will occur. The fact that B has occured has no effect on the probability of A occuring.

76

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

NOTE: If A and B are independent with $Pr[A] \neq 0$, then

ALSO $$Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{Pr[A] \cdot Pr[B]}{Pr[A]} = Pr[B]$$

So the probability that B will occur given that A will occur is exactly the probability that B will occur. The fact that A has occured has no effect on the probability of B occuring.

77

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$Pr[A \cap B] = Pr[A] \cdot Pr[B]$$

NOTE: If A and B are independent with $0 < Pr[A] < 1$, then

$$Pr[B|A^c] = Pr[B] \qquad Pr[B^c|A] = Pr[B^c]$$

We won't prove this now, but it should seem reasonable, it two events are independent, then they should be independent of the complements of one another.

78

EXAMPLE: You buy a tape deck from a supplier. 8% of this supplier's tape deck stock is defective. You buy audio tapes from a supplier. 2% of this stock is defective. What is the probability that you bought a defective tape deck and defective audio tapes?

79

EXAMPLE: You buy a tape deck from a supplier. 8% of this supplier's tape deck stock is defective. You buy audio tapes from a supplier. 2% of this stock is defective. What is the probability that you bought a defective tape deck and defective audio tapes?

$Pr[DTD] = .08 \qquad Pr[DAT] = .02$

$Pr[DTD \cap DAT] = Pr[DTD]Pr[DAT] = .08 \times .02 = \underline{.0016}$

80