

CONDITIONAL PROBABILITY AND INDEPENDENCE:

CONDITIONAL PROBABILITY:

This involves calculating probabilities given some sort of extra information.

ERASE



1



Lecture 10

CONDITIONAL PROBABILITY AND INDEPENDENCE:

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EXAMPLE: 1) A hat contains 3 slips of paper, 2 white and 1 black. You remove, at random, one slip of paper, but DON'T look at it. What is the probability that you got a white slip?

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Answer: $\Pr[\text{You drew a white}] = 2/3$

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2) Next, someone draws a slip from the hat at random from the two remaining slips, and it is white. Now, what is the probability that you got a white slip?

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2) Next, someone draws a slip from the hat at random from the two remaining slips, and it is white. Now, what is the probability that you got a white slip?

Answer: Your situation is the same as if, in the beginning, a white slip was drawn out (rather than after your draw). So, effectively, from a white slip and a black slip, you select one at random. $\Pr[\text{You drew a white}] = 1/2$

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What is the difference between 1) and 2)?

$\Pr[\text{You drew a white}] = 1/2$

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7



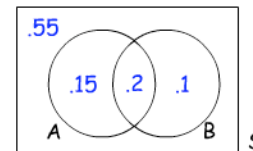
2) Next, someone draws a slip from the hat at random from the two remaining slips, and it is white. Now, what is the probability that you got a white slip?

Answer: You are given additional information. You are told the results of another draw.

$\Pr[\text{You drew a white}] = 1/2$

CONDITIONAL PROBABILITY: Venn diagram approach

Let A and B be subsets of a sample space S . Here's a Venn diagram which includes the probabilities:



This is just an example. Obviously, the actual numbers may vary.

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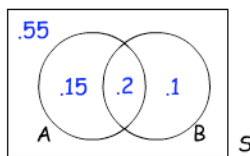


8



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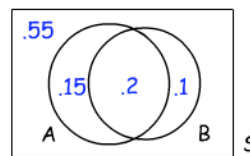
You could, if you were careful, draw the diagram so that S has area 1 (in what ever units you are using), and the sets have each have an area equal to the probabilities shown in the diagram.

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9

CONDITIONAL PROBABILITY: Venn diagram approach

Let A and B be subsets of a sample space S . Here's a Venn diagram which includes the probabilities:



Now drawn to scale.

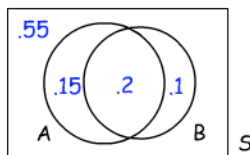
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10

Lecture 10

CONDITIONAL PROBABILITY: Venn diagram approach

Let A and B be subsets of a sample space S . Here's a Venn diagram which includes the probabilities:



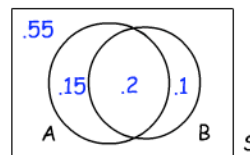
Now pretend that you have magic darts, such that when thrown always hit S and land randomly on S . Then the probability of hitting A on any one toss is .35, and the probability of hitting B is .3, and the probability of hitting $A \cap B$ is .2. The probability of missing A and B is .55.

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11

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B . What is the probability it hit A ?

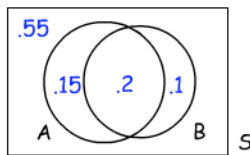


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12

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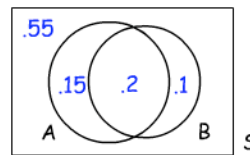
The dart is assumed to be one of the magic darts, that it is just as likely to hit one point in S as another. However, in this case, a reliable witness tells you that it hit B . This is all you know other than it landed "at random."

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13

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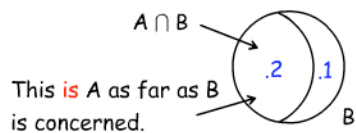
Answer: You are just as likely to hit any given point in B as any other point. It is as if B has become the entire sample space (i.e. THE ENTIRE DART BOARD), and the dart lands randomly on this new board.

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14

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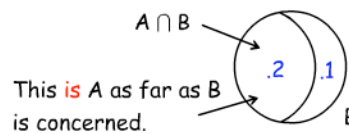
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15

CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B . What is the probability it hit A ?



The entire new "dart board" has area .3. The set $A \cap B$ has area .2.

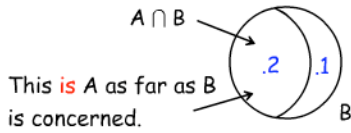
So A accounts for $\frac{.2}{.3} = \frac{2}{3}$ of this new dart board.

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16

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The entire new "dart board" has area .3. The set $A \cap B$ has area .2.

So A accounts for $\frac{.2}{.3} = \frac{2}{3}$ of this new dart board.

If a dart hits at a random place on B, then the probability that it hits A is 2/3.

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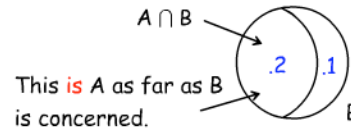


17



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The entire new "dart board" has area .3. The set $A \cap B$ has area .2.

So A accounts for $\frac{.2}{.3} = \frac{2}{3}$ of this new dart board.

If a dart hits at a random place on B, then the probability that it hits A is 2/3. **ANSWER: 2/3**

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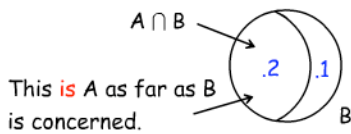
18



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CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



Notice that the answer 2/3 is

$$\frac{\text{area}(A \cap B)}{\text{area}(B)} = \frac{\text{Pr}(A \cap B)}{\text{Pr}(B)}$$

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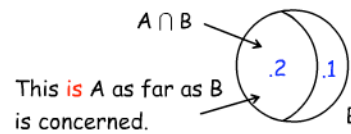


19



CONDITIONAL PROBABILITY: Venn diagram approach

Question: Suppose you throw a dart, don't see where it lands, but are told that it hit B. What is the probability it hit A?



Notice that the answer 2/3 is

$$\frac{\text{area}(A \cap B)}{\text{area}(B)} = \frac{\text{Pr}(A \cap B)}{\text{Pr}(B)} = \text{Pr}[A|B]$$

The official language for the probability of hitting A knowing that you hit B is $\text{Pr}[A|B]$.

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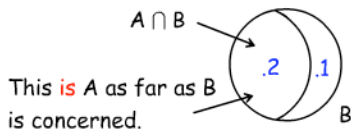


20



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The official language for the probability of hitting A knowing that you hit B is $\text{Pr}[A|B]$. We now have:

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21



CONDITIONAL PROBABILITY:

Definition: Let A and B be subsets of a sample space S. Provided $\text{Pr}[B] \neq 0$, then the probability that A occurs given that B has occurred is defined as:

$$\text{Pr}[A|B] = \frac{\text{Pr}(A \cap B)}{\text{Pr}(B)}$$

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22



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$$\text{Pr}[A|B] = \frac{\text{Pr}(A \cap B)}{\text{Pr}(B)}$$

Effectively this is

$$\text{Pr}[A|B] = \frac{\text{area}(A \cap B)}{\text{area}(B)}$$

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23



EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

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24



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SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white.

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25



EXAMPLE: A hat contains 2 white slips of paper and one black. Two slips of paper are drawn out at random, without replacement. What is the probability that the first slip drawn is white given that the second slip drawn is white?

SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white.

Usually, you don't even bother to write out the sample space, but in this case here it is: (slips W , W^* , and B are in the hat)

$$S = \{(W, W^*), (W^*, W), (W, B), (B, W), (W^*, B), (B, W^*)\}$$

$$n(S)=6 \quad n(W_1) = 4 \quad n(W_1 \cap W_2) = n\{(W, W^*), (W^*, W)\} = 2$$

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26



Lecture 10

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SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = ?$$

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27



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SOLUTION: Let W_1 be the event that the first slip drawn out is white. Let W_2 be the event that the second slip drawn out is white. The problem asks:

$$\Pr[W_1|W_2] = \frac{\Pr(W_1 \cap W_2)}{\Pr(W_2)}$$

ERASE



28



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Finding the numerator is equivalent to solving:

If 2 slips are drawn, one after another, without replacement what is the probability of drawing a white and then a white.

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29



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$$\Pr[W_1|W_2] = \frac{\Pr(W_1 \cap W_2)}{\Pr(W_2)}$$

$$\Pr(W_1 \cap W_2) = \Pr[\text{drawing 2 whites}] = \frac{\# \text{ ways to draw 2 W}}{\# \text{ ways to draw any 2}}$$

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30



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ERASE



31



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ERASE



32



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Denominator: $\Pr(W_2)$

ERASE



33



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Denominator: $\Pr(W_2) = 2/3$

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34



Lecture 10

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35



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37



EXAMPLE: Suppose E and F are events in a sample space S with $\Pr[E] = .65$, $\Pr[F] = .4$ and $\Pr[E \cup F] = .75$. Find
a) $\Pr[E|F]$
b) $\Pr[E|F^c]$

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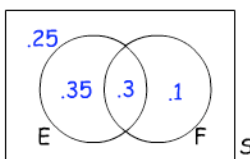
38



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SOLUTION: Earlier, the Venn diagram for this set up was found.

Venn Diagram:



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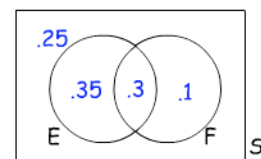
39



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SOLUTION:

$$\Pr[E|F] = \frac{\Pr[E \cap F]}{\Pr[F]}$$



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40

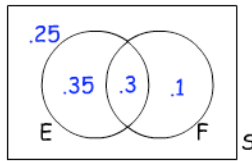


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- $\Pr[E|F]$
- $\Pr[E|F^c]$

SOLUTION:

$$\Pr[E|F] = \frac{\Pr[E \cap F]}{\Pr[F]} = \frac{.3}{.4} = \frac{3}{4}$$



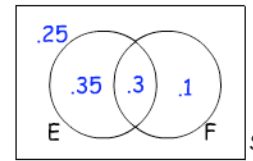
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SOLUTION:

$$\Pr[E|F^c] =$$



41

42

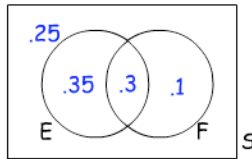
Lecture 10

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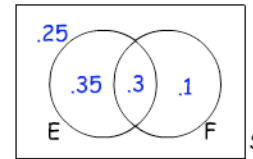
43

EXAMPLE: Suppose E and F are events in a sample space S with $\Pr[E] = .65$, $\Pr[F] = .4$ and $\Pr[E \cup F] = .75$. Find

- $\Pr[E|F]$
- $\Pr[E|F^c]$

SOLUTION:

$$\Pr[E|F^c] = \frac{\Pr[E \cap F^c]}{\Pr[F^c]} = \frac{.35}{.6} = \frac{7}{12}$$



44

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

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SOLUTION:

- A1R = event of getting at least one red martian
- 2R = event of getting two red martians

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SOLUTION:

- A1R = event of getting at least one red martian
- 2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]}$$

SOLUTION:

- A1R = event of getting at least one red martian
- 2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]}$$

$$\Pr[2R \cap A1R] = ? \quad 2R \cap A1R = ?$$

47

48

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49



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A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]}$$

$$\Pr[2R \cap A1R] = ? \quad 2R \cap A1R = 2R$$

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50



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A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]}$$

$$\Pr[2R \cap A1R] = ? \quad 2R \cap A1R = 2R$$

Lecture 10

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51



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A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]}$$

$$\Pr[2R] =$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE



52



SOLUTION:

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2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]}$$

$$\Pr[2R] = \frac{\# \text{ ways to select 2 reds}}{\# \text{ ways to select any 2}}$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE



53



SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]}$$

$$\Pr[2R] = \frac{\# \text{ ways to select 2 reds}}{\# \text{ ways to select any 2}} = \frac{C(3,2)}{C(5,2)}$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE



54



SOLUTION:

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$$\Pr[2R] = \frac{\# \text{ ways to select 2 reds}}{\# \text{ ways to select any 2}} = \frac{C(3,2)}{C(5,2)} = 3/10$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE



55



SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{\Pr[A1R]}$$

$$\Pr[2R] = \frac{\# \text{ ways to select 2 reds}}{\# \text{ ways to select any 2}} = \frac{C(3,2)}{C(5,2)} =$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE



56



SOLUTION:

A1R = event of getting at least one red martian
2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{\Pr[A1R]}$$

$$\Pr[A1R] =$$

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

SOLUTION:

A1R = event of getting at least one red martian
 2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{\Pr[A1R]}$$

select no red
 $\Pr[A1R] = 1 - \Pr[NR]$
 complementary events



57



Lecture 10

EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

SOLUTION:

A1R = event of getting at least one red martian
 2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{\Pr[A1R]}$$

$$\Pr[A1R] = 1 - \Pr[NR] = 1 - \frac{\text{\# ways 2 green}}{\text{\# ways to select any 2}}$$



58



EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

SOLUTION:

A1R = event of getting at least one red martian
 2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{\Pr[A1R]}$$

$$\Pr[A1R] = 1 - \Pr[NR] = 1 - \frac{\text{\# ways 2 green}}{\text{\# ways to select any 2}} = 1 - \frac{1}{10} = 9/10$$



59



EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

SOLUTION:

A1R = event of getting at least one red martian
 2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{9/10}$$

$$\Pr[A1R] = 1 - \Pr[NR] = 1 - \frac{\text{\# ways 2 green}}{\text{\# ways to select any 2}} = 1 - \frac{1}{10} = 9/10$$



60



EXAMPLE: A space ship holds 3 red martians and 2 green martians. Two of the martians are selected at random, without replacement. What is the probability of getting 2 red martians given that you got at least one red martian?

ERASE

SOLUTION:

A1R = event of getting at least one red martian
 2R = event of getting two red martians

$$\Pr[2R|A1R] = \frac{\Pr[2R \cap A1R]}{\Pr[A1R]} = \frac{\Pr[2R]}{\Pr[A1R]} = \frac{3/10}{9/10} = 1/3$$



61



CONDITIONAL PROBABILITY AND INDEPENDENCE:

ERASE

INDEPENDENCE:

This is closely related to conditional probability.



62



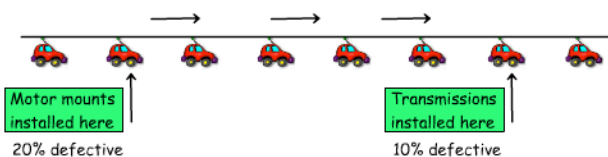
CONDITIONAL PROBABILITY AND INDEPENDENCE:

ERASE

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



These parts come from the supplier and the defects are NOT visible. So the workers don't know they are installing a defective part or not. Whether or not a car gets a defective transmission has nothing to do with whether or not it got defective motor mounts.



63



CONDITIONAL PROBABILITY AND INDEPENDENCE:

ERASE

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.



64

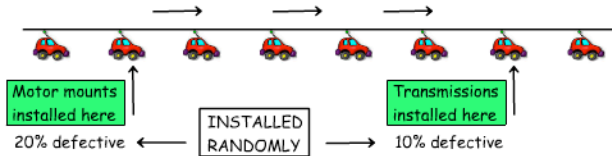


CONDITIONAL PROBABILITY AND INDEPENDENCE:

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20%

ERASE



65

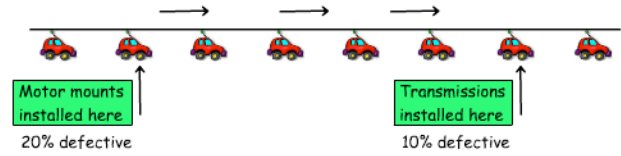


CONDITIONAL PROBABILITY AND INDEPENDENCE:

INDEPENDENCE:

This is closely related to conditional probability.

EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $.1 \times .2 = .02 \Rightarrow 2\%$

ERASE

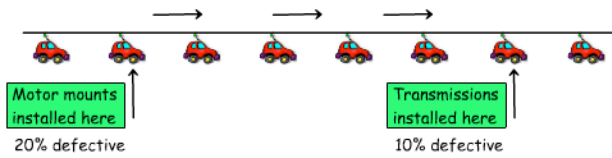


66



Lecture 10

EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $.1 \times .2 = .02 \Rightarrow 2\%$

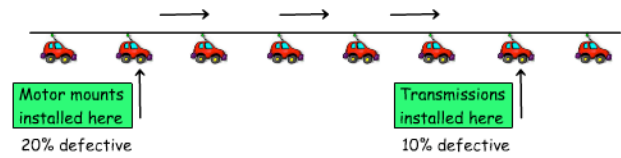
ERASE



67



EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $.1 \times .2 = .02 \Rightarrow 2\%$

RESTATED: If a car is selected at random, let

DM = event of selecting a car with defective motor mounts

DT = event of selecting a car with a defective transmission

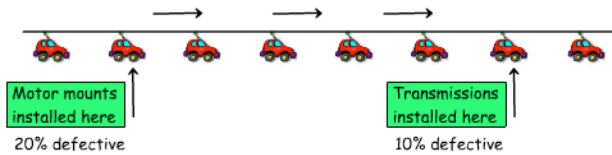
ERASE



68



EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $.1 \times .2 = .02 \Rightarrow 2\%$

RESTATED: $\Pr[DM] = .2$ and $\Pr[DT] = .1$

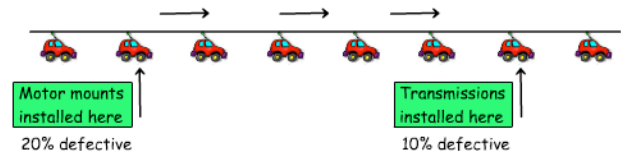
ERASE



69



EXAMPLE: Here is the production line at the Yugo factory:



QUESTION: What % of the cars have a defective transmission and defective motor mounts.

ANSWER: 10% of 20% $.1 \times .2 = .02 \Rightarrow 2\%$

RESTATED: $\Pr[DM] = .2$ and $\Pr[DT] = .1$ and

$\Pr[DM \cap DT] = \Pr[DM] \cdot \Pr[DT] = .1 \times .2 = .02$

event of a defective trans. and motor mounts

ERASE



70



Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

This happens any time the two sets have "nothing to do with one another" as in the previous example.

ERASE



71



Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

This happens any time the two sets have "nothing to do with one another" as in the previous example.

ERASE



72



Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

NOTE: If A and B are independent with $\Pr[B] \neq 0$, then

$$\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$$

ERASE



73



Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

NOTE: If A and B are independent with $\Pr[B] \neq 0$, then

$$\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]} = \frac{\Pr[A] \cdot \Pr[B]}{\Pr[B]} = \Pr[A]$$

ERASE



74



Lecture 10

Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

NOTE: If A and B are independent with $\Pr[B] \neq 0$, then

$$\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]} = \frac{\Pr[A] \cdot \Pr[B]}{\Pr[B]} = \Pr[A]$$

So the probability that A will occur given that B will occur is exactly the probability that A will occur. The fact that B has occurred has no effect on the probability of A occurring.

ERASE



75



Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

NOTE: If A and B are **independent** with $\Pr[B] \neq 0$, then

$$\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]} = \frac{\Pr[A] \cdot \Pr[B]}{\Pr[B]} = \Pr[A]$$

So the probability that A will occur given that B will occur is exactly the probability that A will occur. **The fact that B has occurred has no effect on the probability of A occurring.**

ERASE



76



Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

NOTE: If A and B are **independent** with $\Pr[A] \neq 0$, then

ALSO
$$\Pr[B|A] = \frac{\Pr[A \cap B]}{\Pr[A]} = \frac{\Pr[A] \cdot \Pr[B]}{\Pr[A]} = \Pr[B]$$

So the probability that B will occur given that A will occur is exactly the probability that B will occur. **The fact that A has occurred has no effect on the probability of B occurring.**

ERASE



77



Definition: Let A and B be subsets of a sample space S. The events A and B are said to be independent provided

$$\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$$

NOTE: If A and B are independent with $0 < \Pr[A] < 1$, then

$$\Pr[B|A^c] = \Pr[B] \qquad \Pr[B^c|A] = \Pr[B^c]$$

We won't prove this now, but it should seem reasonable, if two events are independent, then they should be independent of the complements of one another.

ERASE



78



EXAMPLE: You buy a tape deck from a supplier. 8% of this supplier's tape deck stock is defective. You buy audio tapes from a supplier. 2% of this stock is defective. What is the probability that you bought a defective tape deck and defective audio tapes?

ERASE



79



EXAMPLE: You buy a tape deck from a supplier. 8% of this supplier's tape deck stock is defective. You buy audio tapes from a supplier. 2% of this stock is defective. What is the probability that you bought a defective tape deck and defective audio tapes?

ERASE



80



$$\Pr[DTD] = .08 \qquad \Pr[DAT] = .02$$

$$\Pr[DTD \cap DAT] = \Pr[DTD]\Pr[DAT] = .08 \times .02 = \underline{.0016}$$