A set is a collection of objects:

Examples:
A bunch of grapes.
A pack of wolves.

The objects are called elements.

Each wolf is one element in the pack.
(i.e. set)

1

Here is a set:

A={ 1,2,3,a,b,c}

This set is called "A"

Set brackets

2

Lecture 1

Here is a set:

ELEMENTS 6 of them

A={ 1,2,3,a,b,c}

This set is called "A"

Set brackets

The commas serve to separate one element from another.

The elements of this set are: 1 b 3 a 2 c

3

Here is a set:

ELEMENTS 6 of them

A={ 1,2,3,a,b,c}

This set is called "A"

Set brackets

MORE
Notation:

$1 \in$	A	1 is an element in the set A
$d \notin$	A	d is an not element in the set A
$1,b \in$	A	1 is an element in the set A b belongs to the set A (same meaning)

4

Here is a set:

ELEMENTS 6 of them

A={ 1,2,3,a,b,c}

This set is called "A"

Set brackets

MORE
Notation:

$1 \in$	A	1 is an element in the set A
$d \notin$	A	d is an not element in the set A
$1,b \in$	A	1 is an element in the set A b belongs to the set A (same meaning)

5

A={ 1,2,3,a,b,c}

7

A={ 1,2,3,a,b,c}

Comments: 1) Order of listing doesn't matter.
2) Repeating an element doesn't matter.

A={ 1,2,3,a,b,c} ={ c,b,3,a,2,c,b,1,1}

Think of it as a list of guests for a party. If you list someone twice, you still only get one person. No matter where they are on the list, they're still invited.

8

A={ 1,2,3,a,b,c}

Comments: 1) Order of listing doesn't matter.
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Think of it as a list of guests for a party. If you list someone twice, you still only get one person. No matter where they are on the list, they're still invited.

8

Here's a list of the people invited to my party, given in set notation:

{ } -- Also denoted by ø and called the null set or the empty set.

Nobody was invited.

Everybody who was invited said it was the best party that they had ever attended.

ERASE

9

Here is a set:

{ { } } = {ø}

HOW MANY ELEMENTS ARE IN THIS SET, AND WHAT ARE THEY?

ERASE

10

Lecture 1

Here is a set:

{ { } } = {ø}

HOW MANY ELEMENTS ARE IN THIS SET, AND WHAT ARE THEY?

ANSWER: ONE ELEMENT   ø = { }

$\emptyset \in$ { { } }

ERASE

11

A={ 1,2,3,a,b,c} BESIDES ROSTER NOTATION THERE IS ALSO DESCRIPTIVE SET NOTATION:

E={ x: x is a real number with $0<x<5$}

(such that)

E IS THE SET OF ALL x SUCH THAT x IS A REAL NUMBER IN BETWEEN 0 AND 5.

E IS THE SET OF REAL NUMBERS IN BETWEEN 0 AND 5.

ERASE

12

A={ 1,2,3,a,b,c}

ANOTHER EXAMPLE OF DESCRIPTIVE SET NOTATION:

Z={ x ∈ A: x is a number} ={ 1,2,3}

DESCRIPTIVE   ROSTER

ERASE

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MORE SET NOTATION & CONCEPTS:

1) SUBSETS
2) UNIONS
3) INTERSECTIONS
4) UNIVERSAL SETS
5) COMPLEMENTS
6) SET DIFFERENCES

ERASE

14

SUBSETS: SUPPOSE YOU HAVE TWO SETS *A* & *C*.

IF EVERY ELEMENT IN *A* IS ALSO IN *C* THEN *A* IS SAID TO BE A SUBSET *C*. THIS IS DENOTED BY WRITING:

$A \subset C$ (*A* IS A SUBSET OF *C* OR *A* IS CONTAINED IN *C*.)

EXAMPLE: *A*={ 1,2,3,a,b,c}

C={ 1,2,3,6,s,4,a,b,c}

ERASE

15

SUBSETS: SUPPOSE YOU HAVE TWO SETS *A* & *C*.

IF EVERY ELEMENT IN *A* IS ALSO IN *C* THEN *A* IS SAID TO BE A SUBSET *C*. THIS IS DENOTED BY WRITING:

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EXAMPLE: *A*={ 1,2,3,a,b,c}

C={ 1,2,3,6,s,4,a,b,c}

EVERY ELEMENT OF *A* IS IN *C*.

ERASE

16

UNIONS: SUPPOSE YOU HAVE TWO SETS A & B.

FROM THESE TWO SETS, A NEW SET CAN BE FORMED, CALLED THE UNION OF A AND B. THIS NEW SET CON ISTS OF ALL ELEMENTS IN A OR B.

$$A \cup B = \{ x : x \in A \text{ OR } x \in B \}$$

EXAMPLE:

$A = \{ 1,2,3,a,b,c \}$

$B = \{ 1,3,6,s,4,a,c \}$

$A \cup B = \{ 1,2,3,4,6,a,b,c,s \}$

DUMP ALL THE ELEMENTS OF A AND B TOGETHER INTO ONE SET.

17

INTERSECTIONS: SUPPOSE YOU HAVE TWO SETS A & B.

FROM THESE TWO SETS, FORM A NEW SET CALLED THE INTERSECTION OF A AND B. THIS NEW SET CONSISTS OF ALL ELEMENTS IN A AND B.

$$A \cap B = \{ x : x \in A \text{ AND } x \in B \}$$

EXAMPLE:

$A = \{ 1,2,3,a,b,c \}$

$B = \{ 1,3,6,s,4,a,c \}$

$A \cap B = \{ 1,3,a,c \}$

THE INTERSECTION CONSISTS ALL THE ELEMENTS IN A AND B.

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Lecture 1

INTERSECTIONS: SUPPOSE YOU HAVE TWO SETS A & B.

FROM THESE TWO SETS, FORM A NEW SET CALLED THE INTERSECTION OF A AND B. THIS NEW SET CONSISTS OF ALL ELEMENTS IN A AND B.

$$A \cap B = \{ x : x \in A \text{ AND } x \in B \}$$

EXAMPLE:

$A = \{ 1,2,3,a,b,c \}$

$B = \{ 1,3,6,s,4,a,c \}$

$A \cap B = \{ 1,3,a,c \}$

THE INTERSECTION CONSISTS ALL THE ELEMENTS IN A AND B. **THIS IS MORE RESTRICTIVE.**

19

UNIVERSAL SETS:

ALMOST ALL CALCULATIONS WILL TAKE PLACE WITHIN A SET THAT IS ACCEPTED TO BE, IN SOME SENSE, THE UNIVERSE.

EXAMPLE: SUPPOSE YOU ROLL A DIE AND RECORD THE RESULT (1,2,3,4,5, OR 6). IN WORKING ON THIS MATHEMATICALLY, YOU MAY AS WELL ASSUME THAT EVERYTHING TAKES PLACE IN THE SET

$$U = \{ 1,2,3,4,5,6 \}$$

CALLED A UNIVERSAL SET OR A SAMPLE SPACE.

20

UNIVERSAL SETS:

THE UNIVERSAL SET IS USUALLY ASSUMED TO BE EVERY POSSIBLE OUTCOME THAT YOU ARE WILLING TO CONSIDER. THERE IS NO REASON TO INCLUDE ALL ELEMENTS OR IDEAS IN THE ACTUAL UNIVERSE. FOR INSTANCE, YOU DON'T NEED TO INCLUDE THE SUB SANDWICH THAT SOMEONE IS EATING OVER AT DAGWOODS OR A SPECK OF DUST ON NEPTUNE. YOU WILL CHOOSE THE UNIVERSAL SET TO INCLUDE THOSE THINGS NEEDED IN ORDER TO SOLVE WHATEVER PROBLEM YOU ARE WORKING ON.

21

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22

COMPLEMENTS:

SUPPOSE U IS A UNIVERSAL SET, WITH $A \subset U$.

THEN A^c IS A NEW SET THAT CONSISTS OF ALL ELEMENTS OF U NOT CONTAINED IN A. THE SET A^c IS CALLED THE COMPLEMENT OF A.

$$A^c = \{ x \in U : x \notin A \}$$

NOTICE THAT A UNIVERSAL SET HAS TO BE SPECIFIED IN ORDER TO DEFINE THE COMPLEMENT.

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SET DIFFERENCES: NOT IN TEXT

LET A AND B BE SETS.

THEN A-B IS A NEW SET THAT CONSISTS OF ALL ELEMENTS IN A BUT NOT IN B.

$$A - B = \{ x \in A : x \notin B \} = \{ x : x \in A \text{ and } x \notin B \}$$

$$= A \cap B^c$$

EXAMPLE:

$$\{ 1,2,3,a,b,c \} - \{ a,2,d,e,f,b \} = \{ 1,3,c \}$$

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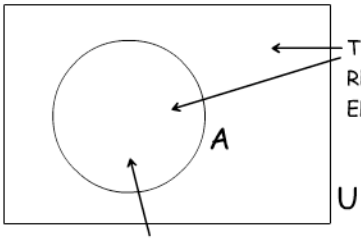
VENN DIAGRAMS:

SETS CAN BE REPRESENTED PICTORIALLY BY THINKING OF EACH POINT ON A PAGE AS AN ELEMENT OF A SET.

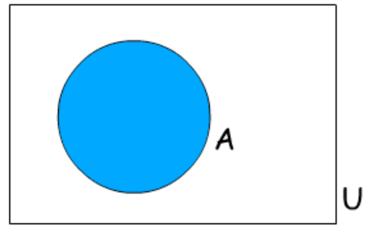


THE POINTS IN HERE REPRESENT THE ELEMENTS OF U

THE POINTS IN HERE REPRESENT THE ELEMENTS OF A.

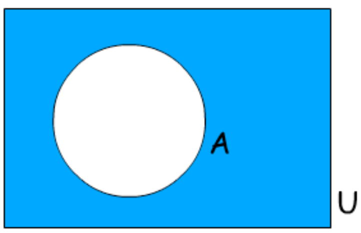
VENN DIAGRAMS:

USUALLY YOU DARKEN IN THE SET OF INTEREST. IF YOU ARE INTERESTED IN THE SET *A* THEN THE VENN DIAGRAM (FOR THE SET *A*) WILL LOOK LIKE:



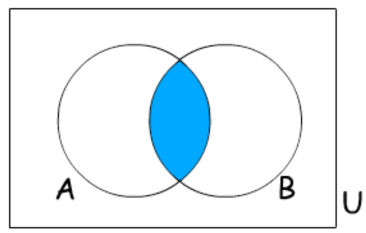
VENN DIAGRAMS:

HERE'S THE VENN DIAGRAM FOR A^c



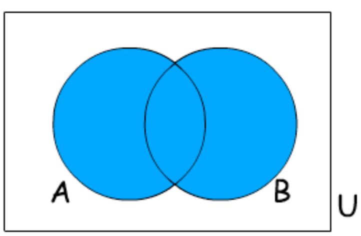
VENN DIAGRAMS:

HERE'S THE VENN DIAGRAM FOR $A \cap B$. IT'S THE OVERLAP OF **A** AND **B**.

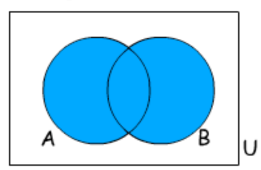


VENN DIAGRAMS:

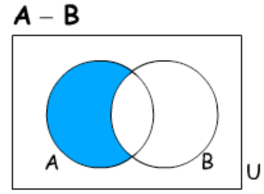
HERE'S THE VENN DIAGRAM FOR $A \cup B$. IT'S ALL OF **A** AND **B** TOGETHER.



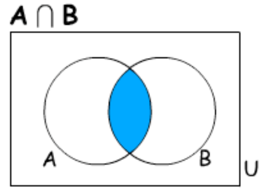
$A \cup B$



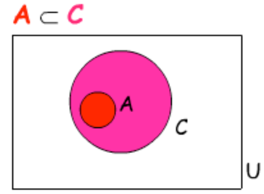
$A - B$



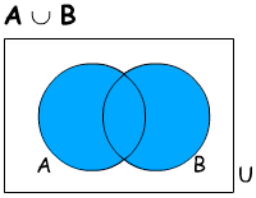
$A \cap B$



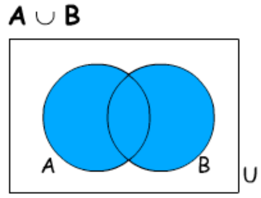
$A \subset C$



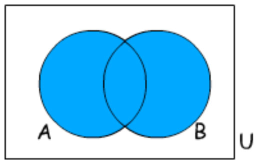
$A \cup B$



$A \cup B$

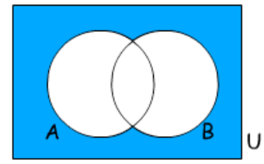


$A \cup B$



WHAT IS THE VENN DIAGRAM FOR SET $(A \cup B)^c$?:

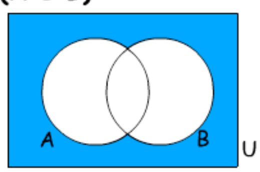
$(A \cup B)^c$



DONE!!!

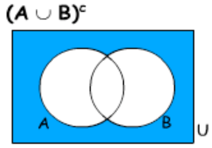
WHAT IS THE VENN DIAGRAM FOR SET $(A \cup B)^c$?:

$(A \cup B)^c$



LET'S PUT THIS ASIDE FOR AWHILE AND WORK ON ANOTHER PROBLEM.

$(A \cup B)^c$



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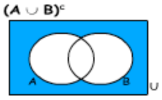
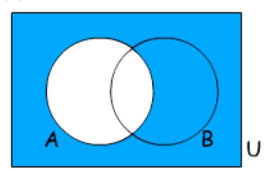
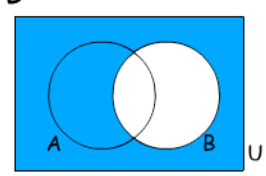


WHAT IS THE VENN DIAGRAM FOR $(A^c \cap B^c)$?

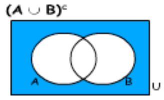
A^c



B^c



$(A \cup B)^c$



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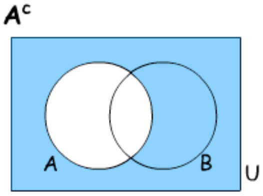
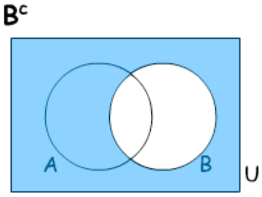
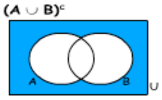


B^c



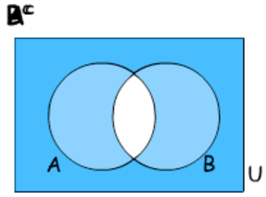
$(A \cup B)^c$



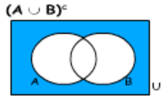
WHAT IS THE VENN DIAGRAM FOR $(A^c \cap B^c)$?

THE OVERLAP REPRESENTS THE SET $A^c \cap B^c$

B^c

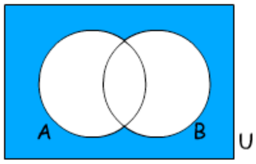


$(A \cup B)^c$

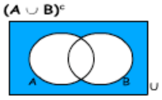


WHAT IS THE VENN DIAGRAM FOR $(A^c \cap B^c)$?

$(A^c \cap B^c)$



$(A \cup B)^c$



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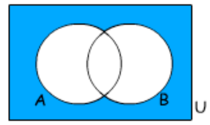
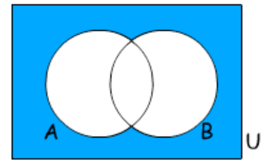
$(A \cup B)^c$



$(A^c \cap B^c)$



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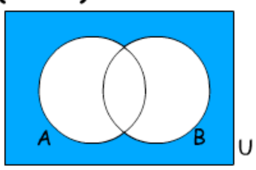
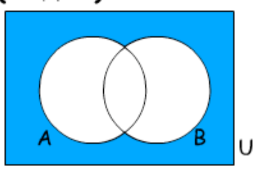
Lecture 1

$(A \cup B)^c$



$(A^c \cap B^c)$



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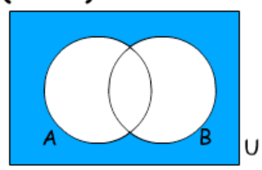
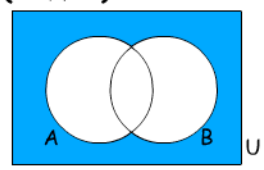
$(A \cup B)^c$



$(A^c \cap B^c)$



THEY ARE THE SAME!!! CONCLUSION:

$(A \cup B)^c = A^c \cap B^c$ deMorgan's Law

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$(A \cup B)^c = A^c \cap B^c$

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$(A \cup B)^c = A^c \cap B^c$

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$(A \cup B)^c = A^c \cap B^c$

THIS CAN BE REWRITTEN AS: ( $\cup$  $)^c$ =  $^c \cap$  ^{c}

WHERE  CAN BE ANY SET AND  CAN BE ANY SET.

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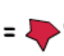
$(A \cup B)^c = A^c \cap B^c$

THIS CAN BE REWRITTEN AS: ( $\cup$  $)^c$ =  $^c \cap$  ^{c}

LET'S SET  $= A^c$ AND  $= B^c$.

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$(A \cup B)^c = A^c \cap B^c$

THIS CAN BE REWRITTEN AS: $(\cup)^c = {}^c \cap {}^c$

LET'S SET $=A^c$ AND $=B^c$.

SO NOW

$(A^c \cup B^c)^c = (A^c)^c \cap (B^c)^c$

ERASE

$(A \cup B)^c = A^c \cap B^c$

THIS CAN BE REWRITTEN AS: $(\cup)^c = {}^c \cap {}^c$

LET'S SET $=A^c$ AND $=B^c$.

SO NOW

$(A^c \cup B^c)^c = (A^c)^c \cap (B^c)^c$

AND

$(A^c \cup B^c)^c = A \cap B$

$(B^c)^c=B$
$(A^c)^c=A$

ERASE

$(A \cup B)^c = A^c \cap B^c$

THIS CAN BE REWRITTEN AS: $(\cup)^c = {}^c \cap {}^c$

LET'S SET $=A^c$ AND $=B^c$.

SO NOW

$(A^c \cup B^c)^c = (A^c)^c \cap (B^c)^c$

AND

$(A^c \cup B^c)^c = A \cap B$

$((A^c \cup B^c)^c)^c = (A \cap B)^c$

TAKE COMPLEMENTS OF BOTH SIDES

ERASE

$(A \cup B)^c = A^c \cap B^c$

THIS CAN BE REWRITTEN AS: $(\cup)^c = {}^c \cap {}^c$

LET'S SET $=A^c$ AND $=B^c$.

SO NOW

$(A^c \cup B^c)^c = (A^c)^c \cap (B^c)^c$

AND

$(A^c \cup B^c)^c = A \cap B$

$((A^c \cup B^c)^c)^c = (A \cap B)^c$

$A^c \cup B^c = (A \cap B)^c$

$)^c)^c =)$

ERASE

$(A \cup B)^c = A^c \cap B^c$

$A^c \cup B^c = (A \cap B)^c$

ERASE

$(A \cup B)^c = A^c \cap B^c$

$A^c \cup B^c = (A \cap B)^c$

ERASE

$(A \cup B)^c = A^c \cap B^c$

$A^c \cup B^c = (A \cap B)^c$

WE NOW HAVE TWO deMorgan's LAWS

THERE ARE OTHER SET IDENTITIES THAT CAN BE DERIVED BY VENN "DIAGRAMOLOGY."

HERES A LIST:

ERASE

SOME SET IDENTITIES:

$(A \cup B)^c = A^c \cap B^c$ deMorgan's

$A^c \cup B^c = (A \cap B)^c$ deMorgan's

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
Distributive Properties

$A - B = A \cap B^c$ This you've seen before.

ERASE

PROBLEM: WHAT IS THE VENN DIAGRAM FOR

$(W \cup C) \cap (W \cap L)^c$?

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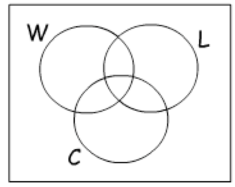
PROBLEM: WHAT IS THE VENN DIAGRAM FOR

$(W \cup C) \cap (W \cap L)^c$?

STEP I: DRAW A VENN DIAGRAM. THIS MAKES SENSE BECAUSE YOU ARE BEING ASKED TO DRAW A VENN DIAGRAM.



DARKEN IN:

$(W \cup C) \cap (W \cap L)^c$

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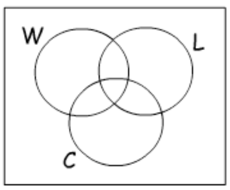
Lecture 1

STEP II: FIGURE IT OUT.

Note that $A \cap B \subset B$. So $(W \cup C) \cap (W \cap L)^c$ is a subset of $(W \cup C)$ and a subset of $(W \cap L)^c$. The set we want consists of everything in $(W \cup C)$ not in $(W \cap L)$. Now, either graph the set directly from this, or graph the overlap of $(W \cup C)$ with $(W \cap L)^c$.

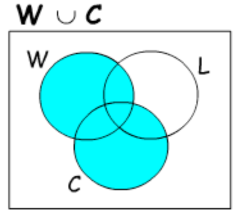


$(W \cup C) \cap (W \cap L)^c$

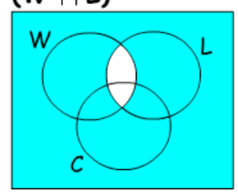
59

To graph the overlap, graph $W \cup C$ and $(W \cap L)^c$ and smash the two sets together:

$W \cup C$



$(W \cap L)^c$



60

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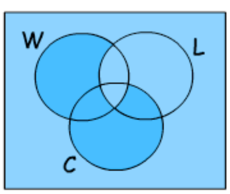

The overlap represents the intersection of $(W \cup C)$ and $(W \cap L)^c$.

62

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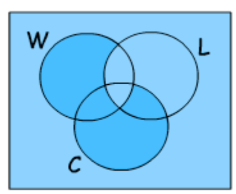
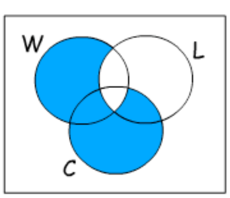


The overlap represents the intersection of $(W \cup C)$ and $(W \cap L)^c$.

62



DONE!!!

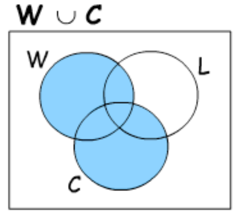
$(W \cup C) \cap (W \cap L)^c$

63

QUESTION: Suppose you wanted to graph $(W \cup C) \cup (W \cap L)^c$ instead of $(W \cup C) \cap (W \cap L)^c$.

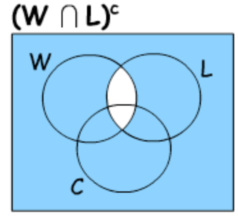
Answer: Smash together and take "everything" not just the overlap.

$W \cup C$



U

$(W \cap L)^c$



64

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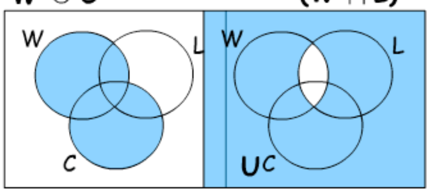
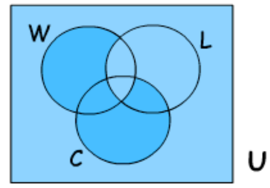
$W \cup C$ $\qquad\qquad$ $(W \cap L)^c$



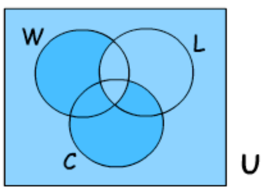
65

SO NOW INCLUDE EVERY POINT THAT IS COLORED BLUE NO MATTER WHAT SHADE.



66

The answer is trivial: It is (all of) **U**.



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COMMENT: The set identities work for 2 or MORE sets:

$(A \cup B)^c = A^c \cap B^c$ $\Leftrightarrow$ $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$

$A^c \cup B^c = (A \cap B)^c$

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ $\Leftrightarrow$

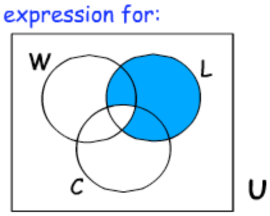
$(A \cap B \cap D) \cup C = (A \cup C) \cap (B \cup C) \cap (D \cup C)$

WHY?

$(A \cup B \cup C)^c = (A \cup (B \cup C))^c = A^c \cap (B \cup C)^c =$
$A^c \cap (B^c \cap C^c) = A^c \cap B^c \cap C^c$

68

Problem: Write an expression for:



69

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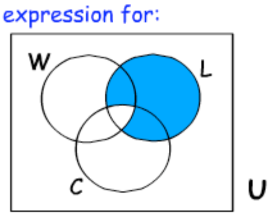


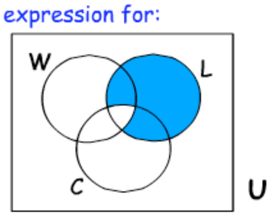
Answer:

1) $L - (W \cap L \cap C)$

There are others.

70

Problem: Write an expression for:



Answer:

1) $L - (W \cap L \cap C)$

There are others. Use set identities to get them.

2) $L \cap (W \cap L \cap C)^c$

$(A \cup B)^c = A^c \cap B^c$ deMorgan's

$A^c \cup B^c = (A \cap B)^c$ deMorgan's

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

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$A - B = A \cap B^c$

71

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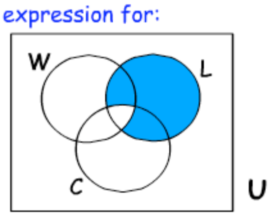

Answer:

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3) $L \cap (W^c \cup L^c \cup C^c)$

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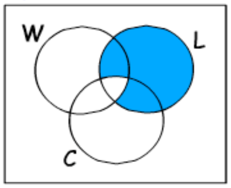

U

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4) $(L \cap W^c) \cup (L \cap L^c) \cup (L \cap C^c)$

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73

Lecture 1

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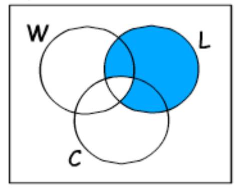

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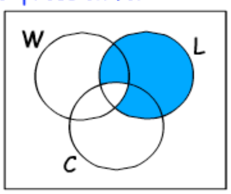
5) $(L \cap W^c) \cup (\emptyset) \cup (L \cap C^c)$

74

Problem: Write an expression for:



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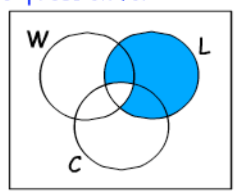

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76

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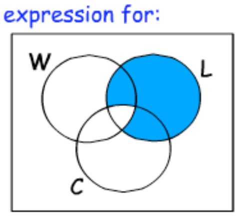

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8) $L \cap (W \cap C)^c$

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CROSS PRODUCTS OF SETS:

IF **A** AND **B** ARE ANY TWO SETS, A NEW SET **AxB** CAN BE FORMED:

$$A \times B = \{ (a,b) : a \in A \text{ and } b \in B \}$$

These are "ordered pairs." The first element belongs to **A**, and the second belongs to **B**.

AxB is the set of all possible ordered pairs with the first element belonging to **A** and the second to **B**.

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CROSS PRODUCTS OF SETS:

EXAMPLE:

Let **A** = { 1,2}. And let **B** = { d,e,f}.

$A \times B$ = { (1,d), (1,e), (1,f), (2,d), (2,e), (2,f)}

A \ B	d	e	f
1	(1,d)	(1,e)	(1,f)
2	(2,d)	(2,e)	(2,f)

USEFUL WAY TO REPRESENT THE ELEMENTS OF **A x B**.

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CROSS PRODUCTS OF SETS:

TRICK QUESTION:

Let **A** = { 1,d}. And let **B** = { d,f}.

Find $A \times B \cap B \times A$.

$A \times B$ = { (1,d), (1,f), (d,d), (d,f)}

$B \times A$ = { (d,1), (f,1), (d,d), (f,d)}

Answer: $A \times B \cap B \times A$ = { (d,d)}.

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FINAL NOTE ABOUT UNIVERSAL SETS:

All sets dealt with are assumed to be a subset of some set U. Why not let a set be ANY collection?

Consider this: How about the collection of all sets? Let

$G = \{ A : A$ is a set$\}$

G is the set of all sets.

81

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Notice that G is strange in that $G \in G$. Well OK... there are sets that are strange.

82

Lecture 1

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Consider next, the set of all NOT strange sets:

$S = \{ A : A$ is a set and $A \notin A\}$.

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Question: Is S strange?

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Question: Is S strange?

1) If S is strange, then $S \in S$. But $S \in S \Rightarrow S$ is NOT STRANGE.

2) If S is NOT strange, then $S \in S$ since S is the collection of all NOT strange sets. But $S \in S \Rightarrow S$ is STRANGE.

85

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CONTRADICTION: $S \in S$ AND $S \notin S$. The easiest way to deal with this is to just not let it happen. One way to do that is to severely restrict what can be a set. And one way to do that is to impose a reasonable (i.e. "small") universal set.

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