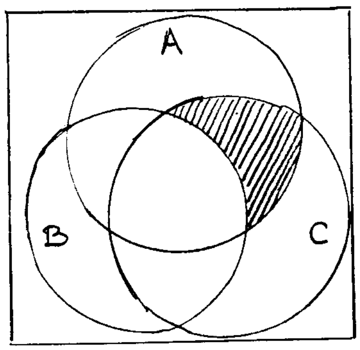
1. **Identify** the shaded set in the following Venn diagram.



(a) $(A \cup B) \cap C'$
(b) $A' \cap B \cap C$
(c) $A \cap B' \cap C'$
(d) $A \cap B' \cap C$
(e) $A' \cap B \cap C'$
(f) none of these

2. One hundred people were interviewed after attending a modern opera theatre performance at the Musical Arts Center. Forty said the middle was tedious, 50 said the ending was beautiful, and 25 said both that the middle was tedious and that the ending was beautiful. How many people neither said that the ending was beautiful, nor said that the middle was tedious?

(a) 30
(b) 35
(c) 40
(d) 45
(e) 50
(f) none of these

3. A box contains 1 red, 2 white and 4 blue balls. Balls are drawn without replacement, noting the color of each, until a blue ball is drawn. **Determine** the number of elements in the sample space for this experiment. [Hint: Draw a tree diagram]

(a) 8
(b) 9
(c) 10
(d) 11
(e) 12
(f) none of these

4. The back room at Bear's Place has 10 tables. Each one seats 4 people. On a night when all the seats are filled, 2 prizes are given away at random to 2 different people. **Determine** the probability that both prizes go to people sitting at the same table.
(a) 3/35
(b) 1/13
(c) 3/43
(d) 3/47
(e) 1/17
(f) none of these

5. A student marketing survey generated the following data

35% like cycling
40% like jogging
25% like bowling
15% like both cycling and jogging
10% like both cycling and bowling
7% like both jogging and bowling
4% like all three.

Find the percentage of those surveyed who like exactly one sport.

(a) 44%	(b) 45%	(c) 46%
(d) 47%	(e) 48%	(f) none of these

6. Events E and F are independent in a sample space S with $Pr[E] = .3$ and $Pr[F] = .5$. **Find** $Pr[E \cup F]$.

(a) .64
(b) .65
(c) .66
(d) .67
(e) .68
(f) none of these

7. Let A and B be events in a sample space with $Pr[A|B] = .5$, and $Pr[A \cap B] = .3$. **Find** $Pr[B]$.

(a) .4
(b) .5
(c) .6
(d) .7
(e) .8
(f) none of these

8. A dorm committee is made up of 3 first-year students and 5 second-year students. A subcommittee of 3 is to be chosen so that there is at least one first-year student and at least one second-year student on the subcommittee. **Find** the number of ways the subcommittee can be chosen.

(a) 40
(b) 45
(c) 63
(d) 70
(e) 120
(f) none of these

9. There are 6 different versions of a final exam. The last row of an auditorium has only 3 students in it. **Find** the number of ways in which the exam can be passed out in the last row so that each student receives a different version of the exam.

(a) 60
(b) 80
(c) 120
(d) 210
(e) 360
(f) none of these

10. A carton contains 4 red, and 6 green apples. Two apples are drawn without replacement, and the color of each is noted. **Determine** the probability that at least one of the apples drawn is green.

(a) $60/90 = 2/3$
(b) $78/90 = 13/15$
(c) $30/42 = 5/7$
(d) $36/42 = 6/7$
(e) $29/32$
(f) none of these

11. In a large fleet of cars, 70% are Fords, and the rest Chevys. Thirty percent of the Fords are equipped with antilock brakes, while only 20% of the Chevys have them. If you insist on a car with antilock brakes, and are given one at random, what is the probability that it will be a Ford?

(a) 7/8
(b) 7/9
(c) 7/11
(d) 7/12
(e) 7/13
(f) none of these

12. Suppose an experiment has a sample space of outcomes $S = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5\}$ with associated weights (probabilities) $w_1 = .20$, $w_2 = .15$, $w_3 = .30$, $w_4 = .25$, and $w_5 = .10$. If $E = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3\}$, **find** $Pr[E']$.

(a) .25
(b) .30
(c) .35
(d) .45
(e) .50
(f) none of these

13. Each day Debbie has a 10% chance of finding a legal parking spot on campus. **Find** the probability that she will find a spot exactly 4 out of 5 days one work week.
(a) $10(.1)^1(.9)^4$
(b) $4(.1)^1(.9)^4$
(c) $4(.1)^4(.9)^1$
(d) $5(.1)^4(.9)^1$
(e) $5(.1)^1(.9)^4$
(f) none of these

14. For \$6 you can have a chance to roll a pair of fair six-sided dice. If you roll a double, i.e. both dice show the same number of dots on top, then you receive back \$30; otherwise you get nothing. What is your expected gain or loss on each play?

(a) lose \$3
(b) lose \$2
(c) lose \$1
(d) gain \$1
(e) gain \$2
(f) none of these

15. A typical M118 student will show up at the correct room for the final exam with probability .8. Suppose that 1300 students take the exam and that afterward, you select 20 at random. **Find** the expected number of students selected who will have gone to the correct room.

(a) 14
(b) 15
(c) 16
(d) 17
(e) 18
(f) none of these

16. **Find** an equation for the straight line which goes through $(1, 2)$ and is parallel to the line $10x + 2y = 1$.

(a) $3x - y = 3$
(b) $5x + y = 9$
(c) $5x + y = 7$
(d) $2x + y = 7$
(e) $5x - y = 3$
(f) none of these

17. Suppose that the cost of a truck rental is related to the number of hours the truck is rented by a linear equation. Also, suppose the cost of a 2-hour rental is \$50 and the cost of a 3-hour rental is \$65. **Find** the cost of a 6-hour rental.

(a) \$80
(b) \$95
(c) \$110
(d) \$120
(e) \$140
(f) none of these

18. Michael's Uptight Cafe makes both low-fat and high-fiber oatmeal cookies. Each low-fat cookie requires 1 ounce of oatmeal and 1/4 ounce of butter, while each high-fiber cookie requires 2 ounces of oatmeal and 1 ounce of butter. Damion, the absent-minded manager, forgets to order cookie ingredients and the cafe has only 24 ounces of oatmeal and 8 ounces of butter. Having fond memories of M118, however, he is able to calculate how many of each type of cookie should be made in order to use all the ingredients. Determine how many low-fat cookies Damion decides to make.
(a) 8
(b) 10
(c) 12
(d) 14
(e) 16
(f) none of these

19. **Determine** which of the following matrices are in reduced form.

$$A = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 1 \\ 0 & 1 & 4 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \quad B = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$C = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \quad D = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

(a) B only
(b) B and D only
(c) A, B and D only
(d) B, C and D only
(e) A and B only
(f) none of these

For the augmented matrices in the next two problems, **determine** which of the following statements is true about the associated system of linear equations:

(a) The system has no solution.
(b) The system has exactly one solution.
(c) The system has exactly two solutions.
(d) The system has infinitely many solutions in which one variable can be selected arbitrarily.
(e) The system has infinitely many solutions in which two variables can be selected arbitrarily.
(f) none of the above.

20. $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 2 & 7 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

21.
$$\left[\begin{array}{ccccc|c} 1 & 4 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Problems 22 and 23 refer to the following matrices

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix} \quad E = \begin{pmatrix} 3 & -1 \end{pmatrix}$$

22. **Determine** which of the following expressions are defined.

(i) AC, (ii) CA, (iii) BC, (iv) DC

(a) (i),(ii),(iii) only
(b) (ii),(iii),(iv) only
(c) (i),(iii),(iv) only
(d) (i),(ii),(iv) only
(e) (i) and (iv) only
(f) none of these

23. **Find** $ED + B$, if defined

(a) $\begin{pmatrix} 1 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & 5 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 2 \end{pmatrix}$ (e) undefined (f) none of these

24. **Find** the value of x for the solution to the system of equations:

$$\begin{aligned} 3x + 9y + 6z &= 6 \\ 2y + 4z &= 2 \\ x + 3y + z &= 2 \end{aligned}$$

(a) -2
(b) -1
(c) 0
(d) 1
(e) 2
(f) none of these

25. **Find** all values of y for the solutions to the following system of equations:

$$\begin{aligned} x + w &= 0 \\ y + z + w &= 3 \\ 2x + z + 4w &= 1 \end{aligned}$$

(a) $y = -3 + 3z$, z arbitrary
(b) $y = 2 - z$, z arbitrary
(c) $y = -3 + 3w$, w arbitrary
(d) $y = 2 + w$, w arbitrary
(e) $y = 2 - z - w$, z arbitrary, w arbitrary
(f) none of these

26. Sally needed to find the inverse to a 3 by 3 matrix A, so she started putting the augmented matrix $[A|I]$ into reduced form. She nearly finished the task, but had to catch a plane home for the summer. This is as far as she worked the problem:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 & 5 & -3 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

Assuming Sally was correct to this point, **find** the (1,2)-entry of A^{-1}.

(a) -2
(b) -1
(c) 4
(d) 5
(e) 6
(f) none of these

27. An economy has two goods: steel and grain. One needs .28 units of steel to produce 1 unit of steel, and .29 units of grain to produce 1 unit of grain. In addition, one needs .3 units of grain to make 1 unit of steel, and .4 units of steel to make one unit of grain. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. **Find** the technology matrix associated with the corresponding Leontief economic model.

(a) $\begin{pmatrix} .28 & .3 \\ .4 & .29 \end{pmatrix}$ (b) $\begin{pmatrix} .72 & -.3 \\ -.4 & .71 \end{pmatrix}$ (c) $\begin{pmatrix} .29 & .4 \\ .3 & .28 \end{pmatrix}$ (d) $\begin{pmatrix} .28 & .4 \\ .3 & .29 \end{pmatrix}$ (e) $\begin{pmatrix} .72 & -.4 \\ -.3 & .71 \end{pmatrix}$

(f) none of these

28. Let

$$A = \begin{pmatrix} .4 & .2 \\ .8 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 20 \\ 80 \end{pmatrix}, \quad \text{and } X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. **Find** x_1. [Recall: $X = AX + D$]

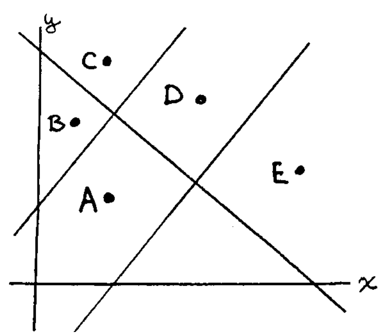
(a) 100
(b) 150
(c) 200
(d) 250
(e) 300
(f) none of these

29. A woodcarver makes toy boats to sell at the Bloomington Art Fair. A simple sailboat requires 1 linear foot of wood and 40 minutes of labor for each boat, while a fancier tugboat requires 2 linear feet of wood, and 90 minutes to make each boat. The carver has 100 linear feet of wood and can spend at most 50 hours making boats between now and the start of the fair. The profit on the sailboat is \$12 per boat, and the profit on the tugboat is \$18 per boat. Let x be the number of sailboats and y the number of tugboats. **Formulate** a linear programming problem to maximize the profit.

(a) Maximize $12x + 18y$ subject to $x + 2y \leq 100$, $40x + 90y \leq 50$, $x \geq 0$, $y \geq 0$
(b) Maximize $18x + 12y$ subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
(c) Maximize $12x + 18y$ subject to $2x + y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
(d) Maximize $12x + 18y$ subject to $2x + y \leq 100$, $90x + 40y \leq 50$, $x \geq 0$, $y \geq 0$
(e) Maximize $12x + 18y$ subject to $x + 2y \leq 100$, $40x + 90y \leq 3000$, $x \geq 0$, $y \geq 0$
(f) none of these

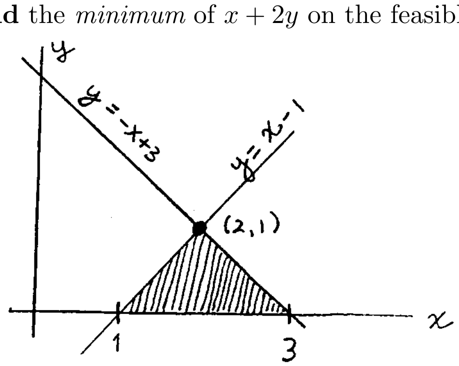
30. **Determine** which point lies in the feasible set given by the constraints

$$x - y \geq -1, \quad x - y \leq 1, \quad x + y \geq 3 \quad x \geq 0, \quad y \geq 0$$



(a) A
(b) B
(c) C
(d) D
(e) E
(f) none of these

31. **Find** the *minimum* of $x + 2y$ on the feasible set shown below.



(a) -3
(b) 1
(c) 0
(d) 3
(e) 4
(f) none of these

32. An M118 student, lost driving through Manhattan, notices that if the traffic light is green as he approaches an intersection, then the light at the next intersection will be green with probability only .2. He also notices that if the light is either yellow or red as he approaches the intersection, the next one will be green with probability .7. **Find** the transition matrix for the associated Markov chain, where state 1 corresponds to having a green light, and state 2 to having a yellow or red light, as the intersection is approached.

(a) $\begin{pmatrix} .2 & .8 \\ .3 & .7 \end{pmatrix}$ (b) $\begin{pmatrix} .7 & .3 \\ .2 & .8 \end{pmatrix}$ (c) $\begin{pmatrix} .8 & .2 \\ .3 & .7 \end{pmatrix}$ (d) $\begin{pmatrix} .8 & .2 \\ .7 & .3 \end{pmatrix}$ (e) $\begin{pmatrix} .2 & .8 \\ .7 & .3 \end{pmatrix}$ (f) none of these

33. Consider the following transition matrix for a Markov chain

$$P = \begin{pmatrix} 1/2 & 1/2 \\ 1/8 & 7/8 \end{pmatrix}.$$

Find the probability of being in state 2 *in the long run.*

(a) 3/4
(b) 3/7
(c) 4/7
(d) 1/5
(e) 4/5
(f) none of these

ANSWERS:

1)D	24)B
2)B	25)D
3)B	26)C
4)B	27)D
5)E	28)A
6)B	29)E
7)C	30)D
8)B	31)B
9)C	32)E
10)B	33)E
11)B	
12)C	
13)D	
14)C	
15)C	
16)C	
17)C	
18)E	
19)B	
20)E	
21)A	
22)C	
23)C	