1) A line runs thru the points $(4, 8)$ and $(9, -2)$.
   It also runs thru $(-1, h)$. What is h?

   a) 5.5

   b) 6.5

   c) 14

   d) 19

   e) •18

   f) None of the above.

$$y = mx + b$$

$$m = \frac{rise}{run} = \frac{-2-8}{9-4} = \frac{-10}{5} = -2$$

$$y = -2x + b$$

$$\Rightarrow 8 = -2 \cdot 4 + b \quad (\text{PLUG IN } x=4, y=8)$$

$$\Rightarrow b = 16$$

$$y = -2x + 16$$

$$\Rightarrow h = (-2)(-1) + 16 = 18$$

---

2) **Find** the value of x for the solution to the following system of equations:

$$3x + 9y + 6z = 15$$
$$0x + 2y + 4z = 2$$
$$1x + 3y + 1z = 5$$

   a) •2

   b) 1

   c) 0

   d) -1

   e) -2

   f) None of the above.

AUGMENTED MATRIX

$$\begin{bmatrix} 3 & 9 & 6 & 15 \\ 0 & 2 & 4 & 2 \\ 1 & 3 & 1 & 5 \end{bmatrix}$$

$\frac{1}{3} R_1$, $\frac{1}{2} R_2$ $\longrightarrow$
$$\begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 5 \end{bmatrix}$$

$-1 R_1 + R_3$ $\longrightarrow$
$$\begin{bmatrix} 1 & 3 & 2 & 5 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow z = 0 \quad \text{So}$$

$$1y + 2z = 1 \Rightarrow y = 1 \text{ and}$$

$$x + 3y + 2z = 5 \Rightarrow x + 3y = 5$$

$$\Rightarrow x + 3 = 5$$

$$\Rightarrow x = 2$$

3) Three (3) Latvian quarters and 7 Latvian nickels have a total weight of 48 grams. Two (2) Latvian quarters and and 5 Latvian nickels have a total weight of 33 grams. How many grams does a Latvian quarter weigh?

All Latvian quarters weight the same amount. All Latvian nickels weigh the same amount.

a) 8

b) 8.5

c) •9

d) 8.75

e) 6

f) None of the above.

$$3Q + 7L = 48$$
$$2Q + 5L = 33 \quad \times 2$$

$\times 3$

$$6Q + 14L = 96$$
$$6Q + 15L = 99$$

$$L = 3$$

So, from the 1st equation.

$$3Q + 7 \cdot 3 = 48$$
$$3Q = 27$$
$$\Rightarrow Q = 9$$

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4) Let

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Let I be the 3×3 identity matrix. Suppose

$$(A - I)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Find x:

a) -2.5

b) -5

c) 3

d) 2

e) •-2

f) None of the above.

$$(A - I) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\Rightarrow z = 3, \quad y = 2, \quad x + z = 1$$
$$\Rightarrow x = -2$$

5) Suppose c and d satisfy the following equation:

$$c\begin{pmatrix} 2 \\ 1 \end{pmatrix} + d\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 19 \\ 27 \end{pmatrix}$$

Find c:

a) 9

b) $-6\frac{1}{4}$

c) 8

d) $6\frac{1}{4}$

e) •6

f) None of the above.

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 19 \\ 27 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 19 \\ 1 & 3 & 27 \end{bmatrix}$$

$$\xrightarrow{-2R_2+R_1} \begin{bmatrix} 0 & -5 & -35 \\ 1 & 3 & 27 \end{bmatrix} \xrightarrow{\frac{1}{5}R_1} \begin{bmatrix} 0 & 1 & 7 \\ 1 & 3 & 27 \end{bmatrix}$$

$$\xrightarrow{-3R_1+R_2} \begin{bmatrix} 0 & 1 & 7 \\ 1 & 0 & 6 \end{bmatrix} \Rightarrow c = 6$$

6) For the augmented matrix below, determine which of the following statements is true about the associated system of linear equations:

(a) The system has no solution.
(b) The system has exactly one solution.
(c) The system has infinitely many solutions in which one variable can be selected arbitrarily.
(d) The system has infinitely many solutions in which two variables can be selected arbitrarily.
(e) none of the above.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & | & 6 \\ 1 & 0 & -2 & 3 & | & 7 \\ 0 & -1 & -3 & 2 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 0 & -2 & 3 & 7 \\ 0 & -1 & -3 & 2 & 2 \end{bmatrix}$$

$$\xrightarrow{-1R_1+R_2} \begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & 2 & 1 \\ 0 & -1 & -3 & 2 & 2 \end{bmatrix}$$

$$\xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{\qquad\qquad}_{\text{NO SOL'N}}$$

$$0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$$

has no solution

7) For the augmented matrix below, determine which of the following statements is true about the associated system of linear equations:

(a) The system has no solution.
(b) The system has exactly one solution.
● (c) The system has infinitely many solutions in which one variable can be selected arbitrarily.
(d) The system has infinitely many solutions in which two variables can be selected arbitrarily.
(e) none of the above.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & | & 5 \\ 1 & 0 & 1 & 1 & | & 6 \\ 1 & 0 & 1 & 2 & | & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 5 \\ 1 & 0 & 1 & 1 & 6 \\ 1 & 0 & 1 & 2 & 4 \end{bmatrix}$$

$\xrightarrow{-1R_2+R_3}$

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 5 \\ 1 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & ① & 1 & 0 & 5 \\ ① & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & ① & -2 \end{bmatrix}$$

↑ FREE

I FREE VARIABLE

---

8) Let A and C each be 2×2 matrices, and suppose

$$CA = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{where} \quad A^{-1} = \begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}.$$

Find the $(2,1)$ entry of C (second row first column).

a) ●4
b) 8
c) 10
d) 6
e) 5
f) None of the above.

$CA = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow C = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} A^{-1}$

$C = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 4 & 1 \end{pmatrix}$

$= \begin{pmatrix} ? & ? \\ ④ & ? \end{pmatrix}$

$0\cdot3 + 1\cdot4 = 4$

9) Here is a system of 2 equations and 4 unknowns:

$$2x_1 + 4x_2 + 4x_3 + 4x_4 = 5$$
$$4x_1 + 8x_2 + 10x_3 + 8x_4 = 12$$

$$\begin{bmatrix} 2 & 4 & 4 & 4 & 5 \\ 4 & 8 & 10 & 8 & 12 \end{bmatrix}$$

IF this system of equations is solved for x_1 in terms of the free variables for this system, and IF all of the free variables are given (i.e. set to) the value 5, then the value for x_1 is:

$-2R_1 + R_2 \longrightarrow \begin{bmatrix} 2 & 4 & 4 & 4 & 5 \\ 0 & 0 & 2 & 0 & 2 \end{bmatrix}$

a) •-19.5

b) $x_1 = -14.5$

c) $x_1 = \frac{5}{2}$.

$-2R_2 + R_1 \longrightarrow \begin{bmatrix} 2 & 4 & 0 & 4 & 1 \\ 0 & 0 & 2 & 0 & 2 \end{bmatrix}$

$\frac{1}{2}R_1 \atop \frac{1}{2}R_2 \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 2 & .5 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

d) $x_1 = 10.5$

e) $x_1 = -12.5$

f) None of the above.

$X_1 = .5 - 2x_2 - 2x_3 = .5 - 2\cdot5 - 2\cdot5$
$= -19.5$

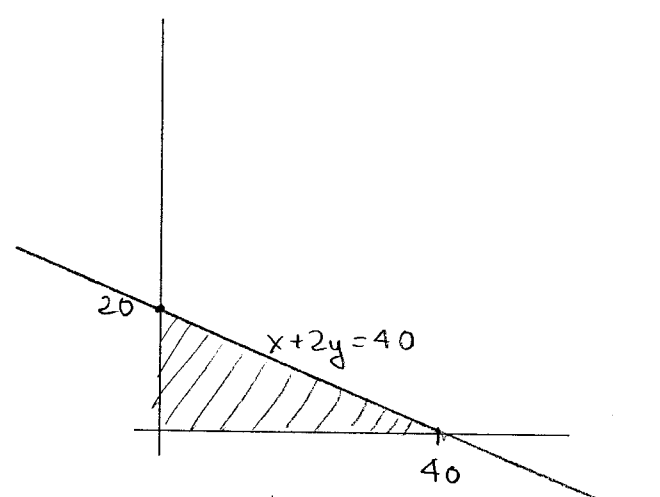10) Find the **maximum** of $2x + 3y$ subject to the constraints

$$\begin{aligned} x + 2y &\le 40 \\ x &\ge 0 \\ y &\ge 0. \end{aligned}$$

a) 120

b) •80

c) 100

d) 60

e) 160

f) None of the above.

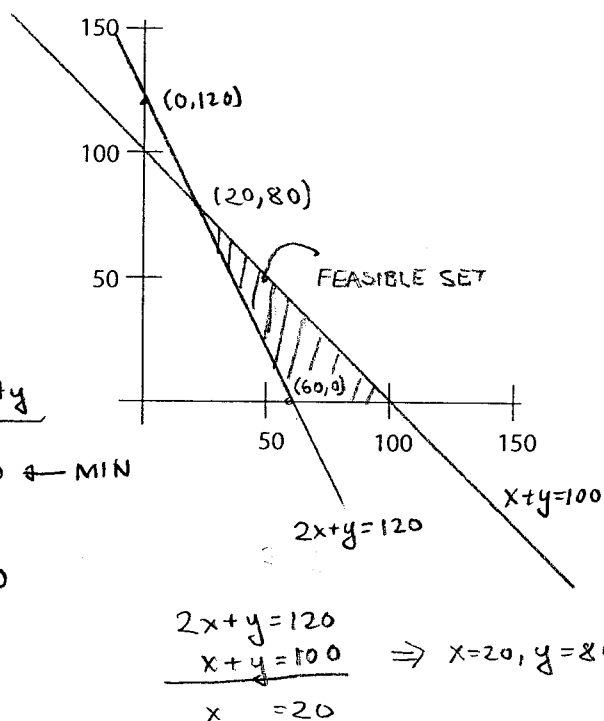| CORNER PTS.	$2x + 3y$
$(0,0)$	0
$(40,0)$	80 ← MAX
$(0,20)$	60

11) Find the **minimum** of $5x + 4y$ subject to the constraints

$$
\begin{aligned}
x + y &\leq 100 \\
2x + y &\geq 120 \\
x &\geq 0 \\
y &\geq 0.
\end{aligned}
$$

Pay close attention to the <u>direction</u> of the inequalities.

a) 420

b) 0

c) ●300

d) 500

e) 400

f) None of the above.

CORNER PTS.	$5x+4y$
(60,0)	300 ←— MIN
(100,0)	500
(20,80)	420

150 (0,120)
100
50 (20,80)
FEASIBLE SET
(60,0)
50 100 150
$x+y=100$
$2x+y=120$

$2x+y=120$
$x+y=100$ ⟹ $x=20, y=80$
_____
$x = 20$

---

12) What is the entry in the 1st row 2nd column of the **inverse** of the matrix given below?

$$
\begin{pmatrix}
0 & 0 & 1 \\
1 & 1 & 0 \\
1 & 2 & 0
\end{pmatrix}
$$

Restatement: Find the (1,2) entry of the inverse.

a) 1/2

b) 1

c) -1

d) ●2

e) -1/2

f) None of the above.

$\left[\begin{array}{ccc|ccc}
0 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 \\
1 & 2 & 0 & 0 & 0 & 1
\end{array}\right]$

$\xrightarrow{-R_1 + R_2}$

$\left[\begin{array}{ccc|ccc}
0 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & -1 & 1
\end{array}\right]$

$\xrightarrow{-R_3 + R_2}$

$\left[\begin{array}{ccc|ccc}
0 & 0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 2 & -1 \\
0 & 1 & 0 & 0 & -1 & 1
\end{array}\right]$

SEND R_1 TO R_3

$\left[\begin{array}{ccc|ccc}
1 & 0 & 0 & 0 & ② & -1 \\
0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 1 & 0 & 0
\end{array}\right]$

Set-up for problems 13 and 14:

A mining company runs two mines - the Lucky Strike and the Red Star. The cost of mining one ton of material from the Lucky Strike mine is $1100. The cost of mining one ton of material from the Red Star mine is $900. Each ton from the Lucky Strike mine yields .75 grams of gold, 40 grams of silver, and 100 grams of copper. Each ton from the Red Star mine yields .5 grams of gold, 50 grams of silver, and 120 grams of copper. This is summarized in the table below:

	Gold	Silver	Copper	Cost
Lucky Strike	.75	40	100	$1100
Red Star	.5	50	120	~~$900~~ $800

The mining company must deliver 1,000 grams of gold, 63,000 grams of silver, and 100,000 grams of copper. Mining agreements dictate that more tonnage has to be mined from the Red Star mine than from the Lucky Strike mine. How many tons should be mined from each mine in order to MINIMIZE total cost and meet the delivery requirements?

13) (5 PTS.) For the linear programming problem corresponding to this set-up, what is the objective function? Let

g_1 = # of grams of gold mined from the Lucky Strike mine
g_2 = # of grams of gold mined from the Red Star mine
s_1 = # of grams of silver mined from the Lucky Strike mine
s_2 = # of grams of silver mined from the Red Star mine
c_1 = # of grams of copper mined from the Lucky Strike mine
c_2 = # of grams of copper mined from the Red Star mine
x = # of tons of material mined from the Lucky Strike mine
y = # of tons of material mined from the Red Star mine

Answer ___ $1100x + 800y$ ___

14) (7 PTS.) What are the constraint equations for this set-up? List them:

$.75x + .5y \geq 1000$ $y \geq 0$

$40x + 50y \geq 63,000$ $y \geq x$

$100x + 120y \geq 100,000$

$x \geq 0$