

1) Find the y-coordinate of the intersection of the lines  $3x - 2y = 11$  and  $5x + y = 1$

a) -1

b) 2

c) 4

d) 1

e) •-4

f) None of the above.

$$\begin{array}{r} 3x - 2y = 11 \longrightarrow 3x - 2y = 11 \\ 5x + y = 1 \xrightarrow{\times 2} \underline{10x + 2y = 2} \\ \hline 13x = 13 \end{array}$$

$$\begin{aligned} x = 1 \Rightarrow 3 \cdot 1 - 2y = 11 &\Rightarrow -2y = 8 \\ &\Rightarrow y = -4 \end{aligned}$$

2) Let

$$\begin{aligned} x + y + z &= 8 \\ x + 2y + z &= 9 \\ x + y + 2z &= 10 \end{aligned}$$

AUGMENTED  
MATRIX

$$\begin{pmatrix} 1 & 1 & 1 & 8 \\ 1 & 2 & 1 & 9 \\ 1 & 1 & 2 & 10 \end{pmatrix}$$

Find  $x$ :

a) -3

b) -4

c)  $5/2$

d) •5

e) 2

f) None of the above.

$$\xrightarrow{-R_1 + R_2} \begin{pmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 10 \end{pmatrix}$$

$$\xrightarrow{-R_1 + R_3} \begin{pmatrix} 1 & 1 & 1 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\Rightarrow z = 2, y = 1, \text{ and}$$

$$\begin{aligned} x + y + z = 8 &\Rightarrow x + 1 + 2 = 8 \\ &\Rightarrow x = 5 \end{aligned}$$

3) Which of these equations describes a line with y-intercept -5 and x-intercept 3?  $\Rightarrow$  line passes thru  $(0, -5), (3, 0)$

a)  $3y = 5x - 15$

b)  $x - 5 = y + 3$

c)  $y = \frac{1}{3}x + 5$

d)  $y = 3x - 56$

e)  $-5y = 3x$

f) None of the above.

$$m = \frac{0 - (-5)}{3 - 0} = \frac{5}{3}$$

$$y = mx + b; \text{ so } y = \frac{5}{3}x + b$$

$$\text{@ } (3, 0), 0 = \frac{5}{3} \cdot 3 + b \Rightarrow b = -5$$

$$y = \frac{5}{3}x - 5$$

$$\Rightarrow 3y = 5x - 15$$

4) Let

column 4 of A

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 2 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \boxed{0 & 0 & 0 & 4 & 0} \end{pmatrix} \text{ row 5 of B}$$

Find the entry in row 5 and column 4 of  $BA + A$ .

a) 14

b) 10

c) 24

d) 19

e) 16

f) None of the above.

$$(BA)_{5,4} = 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 4 \cdot 3 + 0 \cdot 2 = 12$$

$$A_{5,4} = 2$$

$$(BA+A)_{5,4} = (BA)_{5,4} + A_{5,4} = 12 + 2 = 14$$

5) It costs \$123 to send 6 children and 10 adults to the theatre. It costs \$100 to send 10 children and 5 adults to the theatre. Assuming that the price for admission for each adult is the same, and that the price for admission for each child is the same, what is the price of admission for one adult? All costs are admission costs. Hint: Set-up two equations with two unknowns.

$$\begin{array}{r}
 6C + 10A = 123 \\
 10C + 5A = 100
 \end{array}
 \begin{array}{l}
 \times 6 \\
 \times 10
 \end{array}$$


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$$\begin{array}{r}
 60C + 100A = 1230 \\
 - (60C + 30A = 600) \\
 \hline
 70A = 630 \\
 \Rightarrow A = 9
 \end{array}$$

- a) \$8.50
- b) \$9.75
- c) •\$9.00
- d) \$10.00
- e) \$9.50
- f) None of the above.

6) Here is a system of 2 equations and 4 unknowns:

$$\begin{array}{l}
 2x_1 + 3x_2 + 3x_3 + 1x_4 = 4 \\
 4x_1 + 7x_2 + 7x_3 + 2x_4 = 9
 \end{array}$$

IF this system of equations is solved\* for  $x_1$  in terms of the free variables for this system, and IF all of the free variables are given (i.e. set to) the value 5, then the value for  $x_1$  is:

\*Here it is assumed that the solution is found by the standard method of forming the augmented matrix and row reducing. Interchanging columns is not allowed.

- a) • $x_1 = -2$
- b)  $x_1 = -31$
- c)  $x_1 = -31/2$
- d)  $x_1 = -1$ .
- e)  $x_1 = 22$
- f) None of the above.

AUGMENTED MATRIX

$$\begin{array}{l}
 \left[ \begin{array}{cccc|c} 2 & 3 & 3 & 1 & 4 \\ 4 & 7 & 7 & 2 & 9 \end{array} \right] \\
 \xrightarrow{-2R_1+R_2} \left[ \begin{array}{cccc|c} 2 & 3 & 3 & 1 & 4 \\ 0 & 1 & 1 & 0 & 1 \end{array} \right] \\
 \xrightarrow{-3R_2+R_1} \left[ \begin{array}{cccc|c} 2 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{array} \right] \\
 \xrightarrow{1/2 R_1} \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1 & 0 & 1 \end{array} \right] \\
 \Rightarrow x_1 + 1/2 x_4 = 1/2 \\
 \Rightarrow x_1 = \frac{1}{2} - \frac{1}{2} x_4 \quad \begin{array}{l} \uparrow \\ x_4 = 5 \end{array} = \frac{1}{2} - \frac{1}{2} \cdot 5 = \underline{\underline{-2}} \\
 \text{If } x_4 = 7, \text{ then the answer is } \frac{1}{2} - \frac{1}{2} \cdot 5 = -3
 \end{array}$$

7) For the augmented matrix below, determine which of the following statements is true about the associated system of linear equations:

- (a) The system has no solution.  
 (b) The system has exactly one solution.  
 (c) The system has infinitely many solutions in which one variable can be selected arbitrarily.  
 (d) The system has infinitely many solutions in which two variables can be selected arbitrarily.  
 (e) none of the above.

$$\left[ \begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 1 & 4 \\ 1 & 1 & 1 & 7 \end{array} \right] \xrightarrow{-R_3+R_4} \left[ \begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 3 \end{array} \right] \xrightarrow{-R_2+R_4} \left[ \begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$\Rightarrow$  B since all 3 columns have pivots  $\neq 0$ .

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 (d) The system has infinitely many solutions in which two variables can be selected arbitrarily.  
 (e) none of the above.

$$\left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 3 & 17 \\ 0 & 2 & 2 & 2 & 12 \end{array} \right]$$

$$\xrightarrow{-R_1+R_2} \left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 2 & 12 \\ 0 & 2 & 2 & 2 & 12 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cccc|c} P & P & F & F & \\ 1 & 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 2 & 12 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

There will be 2 pivots in the final reduced form  $\Rightarrow$  2 free variables

$\Rightarrow$  D

9) What is the entry in the 1<sup>st</sup> row 2<sup>nd</sup> column of the inverse of the matrix given below?

$$\begin{pmatrix} 1/3 & 1 \\ 0 & -3 \end{pmatrix}$$

$$\left( \begin{array}{cc|cc} \frac{1}{3} & 1 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{array} \right)$$

Restatement: Find the (1,2) entry of the inverse.

a) •1

b) 1/2

c) -1

d) -1/2

e) 2

f) None of the above.

$$\xrightarrow{3R_1} \left( \begin{array}{cc|cc} 1 & 3 & 3 & 0 \\ 0 & -3 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{1 \cdot R_2 + R_1} \left( \begin{array}{cc|cc} 1 & 0 & 3 & 1 \\ 0 & -3 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{-1/3 R_2} \left( \begin{array}{cc|cc} 1 & 0 & 3 & 1 \\ 0 & 1 & 0 & -1/3 \end{array} \right) \begin{array}{l} \leftarrow \text{ROW 1} \\ \uparrow \\ \text{COLUMN 2} \end{array}$$

10) Find the maximum of  $4x + y$  subject to the constraints

$$\begin{aligned} x + y &\leq 100 \\ 5x + y &\leq 200 \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$

a) 220

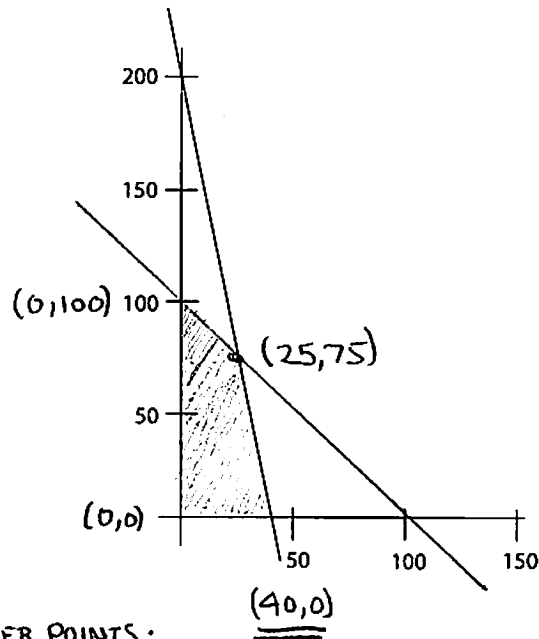
b) •175

c) 160

d) 400

e) 200

f) None of the above.



CORNER POINTS:

$$5x + y = 200 \text{ @ } y=0 \Rightarrow 5x = 200 \Rightarrow \underline{\underline{x=40}}$$

$$\begin{array}{r} -(x + y = 100) \\ 5x + y = 200 \\ \hline 4x = 100 \end{array}$$

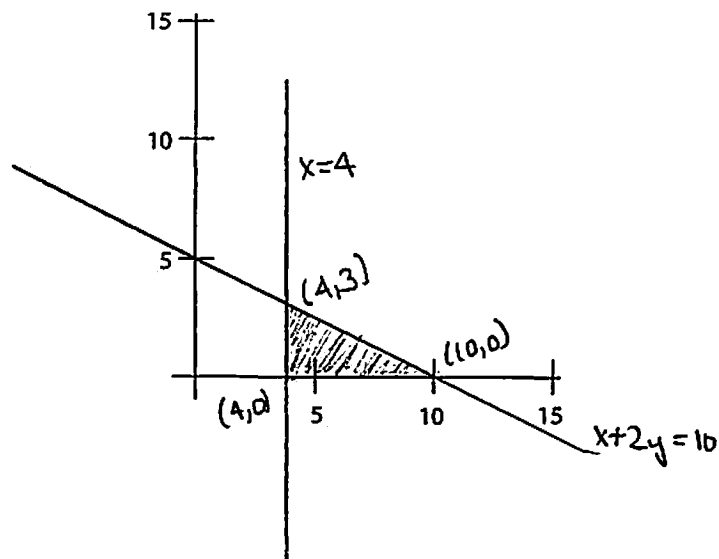
$$\Rightarrow x=25 \Rightarrow y=75 \Rightarrow (25,75)$$

	$4x + y$
(0,0)	0
(0,100)	100
(40,0)	160
(25,75)	175 ← MAX

11) Find the minimum of  $10x - y$  subject to the constraints

$$\begin{aligned} x + 2y &\leq 10 \\ x &\geq 4. \\ y &\geq 0. \end{aligned}$$

- a) 63
- b) •37
- c) 40
- d) 5
- e) 100
- f) None of the above.



$$x + 2y = 10, x = 4 \Rightarrow y = 3 \Rightarrow (4, 3)$$

	$10x - y$
$(4, 0)$	40
$(4, 3)$	37 ← MIN
$(10, 0)$	100

12) Let

$$A = \begin{pmatrix} 2 & 0 \\ 7 & 5 \end{pmatrix}.$$

$$A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = A^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

Find  $x$  where

MULTIPLY BOTH SIDES ON LEFT BY  $A$ .

$$A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = A^{-1} \begin{pmatrix} x \\ y \end{pmatrix}.$$

$$A A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = A A^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Think!!

- a) 5
- b) 7
- c) 10
- d) 1/10
- e) •4
- f) None of the above.

$$\Rightarrow A^2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow \text{1ST COLUMN OF } A^2 = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & 0 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 7 & 5 \end{pmatrix} = \begin{pmatrix} 4 & \star \\ \star & \star \end{pmatrix} \Rightarrow x = 4$$

↑  
1ST COLUMN

**Set-up for problems 13 and 14:**

The Ziggity Bakery produces the Super Cookie and the Deluxe Cookie. The Super Cookie contains 5 chocolate chips and 3 macadamia nuts. The Deluxe Cookie contains 4 chocolate chips and 1 macadamia nut. The sale of each Super Cookie yields a profit of 75 cents. The sale of each Deluxe Cookie yields a profit of 25 cents. The bakery will sell all cookies made. For the next production run, the bakery has available 650 chocolate chips and 200 macadamia nuts, and more than an ample supply of all other ingredients used (e.g. flour, sugar, etc.). How many of each type of cookie should be made in order to maximize profits?

In setting up this problem let,

$x$  = # number of Super Cookies produced

$y$  = # number of Deluxe Cookies produced

13) For the linear programming problem corresponding to this set-up, what is the objective function:

- a)  $5x + 4y$
- b)  $x+y$
- c)  $75x + 25y$
- d)  $3x + y$
- e)  $650x + 200y$
- f) None of the above.

14) Two of the equations listed below are constraint equations that would be used in the set-up of this problem. Write BOTH letters corresponding to these two equations on your answer sheet. You have to get both letters correct in order to get any credit.

a)  $110x + 150y \leq 26,400$

b)  $x \geq 0$  # SUPER COOKIES  $\geq 0$ .

c)  $5x + 4y \leq 650$  # CHOC. CHIPS  $\leq 650$

d)  $3x + y \geq 200$

e)  $75x + 25y \leq 200$

f)  $75x + 25y \geq 650$