1. What is the value of x at the intersection of the line $2x + 3y = 8$ and $-x + 2y = 3$.

$$\begin{aligned} 2x+3y &= 8 \\ -x+2y &= 3 \end{aligned} \xrightarrow{\times 2} \begin{aligned} 2x+3y &= 8 \\ -2x+4y &= 6 \\ \hline 7y &= 14 \end{aligned}$$

$\Rightarrow y=2$. SUBSTITUTE INTO EITHER EQ: $2x+3(2)=8$

$\Rightarrow 2x = 2$

$\Rightarrow \underline{\underline{x=1}}$

2. Let $a \neq 0$. Find the entry in the second row and third column of the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3a \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3a & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-3aR_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -3a \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

ONLY ROW 1 WILL BE AFFECTED IN ANY FURTHER ROW OPS.

ANSWER: $\underline{\underline{-3a}}$

3. Find an equation for the straight line which goes through $(1, 9)$ and is parallel to the line $6x - 3y = 8$.

(a) $3y = x + 26$

(b) $y = 2x + 7$

(c) $y = 7x + 2$

(d) $6x - 3y = 7$

(e) none of the above.

$$6x - 3y = 8 \Rightarrow 6x - 8 = 3y \Rightarrow 2x - 8/3 = y$$

$\Rightarrow$ SLOPE = 2.

EQ. OF LINE HAS FORM

$$y = 2x + b$$

FEED IN (1,9):

$$9 = 2 \cdot 1 + b \Rightarrow b = 7$$

SO

$$\underline{\underline{y = 2x + 7}}$$

4. Find the entry in the second row and third column of the reduced form of the following matrix:

$$\begin{bmatrix} 1 & 1 & 5 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 5 \\ -1 & 1 & -1 \end{bmatrix} \xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & 1 & 5 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{\frac{1}{2} R_2} \begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 2 \end{bmatrix}$$

COULD STOP HERE

$$\xrightarrow{-R_2 + R_1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

2nd ROW, 3rd COLUMN

ANSWER: 2

For each of the augmented matrices in the next three problems, determine which of the following statements is true about the associated system of linear equations:

(a) The system has no solution.

(b) The system has exactly one solution.

(c) The system has infinitely many solutions in which one variable can be selected arbitrarily.

(d) The system has infinitely many solutions in which two variables can be selected arbitrarily.

(e) none of the above.

5. $$\left[\begin{array}{rrr|r} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & 4 \\ 1 & 2 & 0 & 12 \\ -1 & -1 & 1 & -7 \end{array}\right] \xrightarrow[R_1+R_4]{-1R_1+R_3} \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & 4 \\ 0 & 2 & 2 & 9 \\ 0 & -1 & -1 & -4 \end{bmatrix} \xrightarrow{-2R_2+R_3} \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & -4 \end{bmatrix}$$

NO SOLUTION (0,0,0,1)
ANSWER: A

6. $$\left[\begin{array}{rr|r} 1 & 2 & 5 \\ -1 & -1 & -4 \\ 2 & 2 & 8 \\ 1 & 3 & 6 \end{array}\right] \xrightarrow[-R_1+R_4]{R_1+R_2,\ -2R_1+R_3} \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow[-R_2+R_4]{2R_2+R_3} \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ EXACTLY ONE SOL'N
ANSWER: B

7. $$\left[\begin{array}{rrrr|r} 1 & 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ -1 & -3 & 1 & -2 & -1 \end{array}\right] \xrightarrow{R_1+R_3} \begin{bmatrix} 1 & 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

PIVOTS

COLUMNS 2 AND 4 REPRESENT FREE VARIABLES

$\Rightarrow$ ANSWER: D

8. Suppose that the cost of a truck rental is related to the number of days the truck is rented by a linear equation. Also, suppose the cost of a 2-day rental is \$90 and the cost of a 5-day rental is \$198. Find the cost of a 9-day rental.

5 DAY	198
2 DAY	90
	108

$\frac{108}{3} = 36 = \text{COST/DAY}.$

ANSWER: $\underbrace{198}_{\text{5 DAYS}} + \underbrace{4 \cdot 36}_{\text{4 MORE DAYS}} = \underline{342}$

LET C = COST/DAY, TC = TOTAL COST.

THEN $TC = Cx + B$ WHERE x = # DAYS RENTAL

SO $90 = C \cdot 2 + B$
$198 = C \cdot 5 + B$ $\Rightarrow 108 = 3C \Rightarrow C = 36$

SO $TC = 36 + B$, BUT $90 = 2 \cdot 36 + B \Rightarrow B = 18$ $TC = 36x + 18$

$\Rightarrow TC = 36 \cdot 9 + 18 = \underline{342}$

9. Suppose that x and y are numbers such that

$$\begin{bmatrix} -1 & x & 2 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 11 & y \\ 3 & 1 \\ x & 0 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 18 & 2 \end{bmatrix}$$

Find x

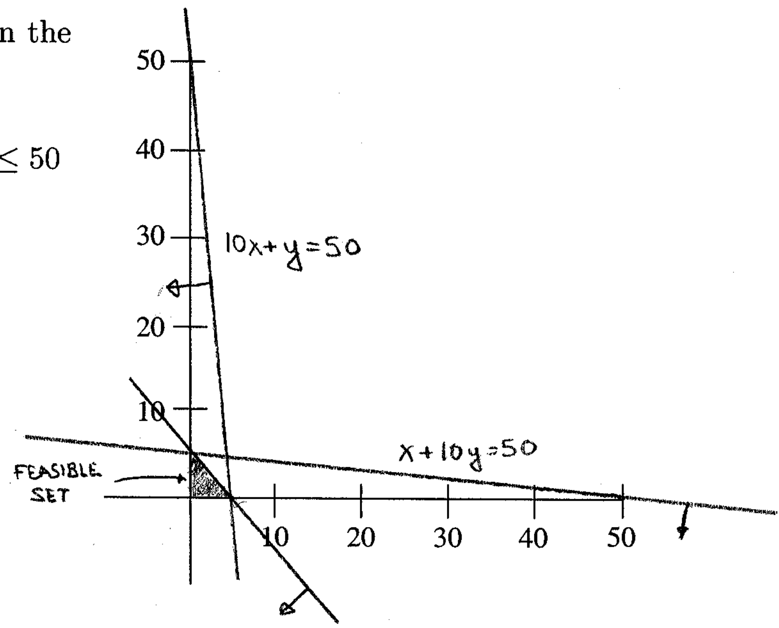
$(-1 \ x \ 2)\begin{pmatrix} 11 \\ 3 \\ x \end{pmatrix} = (9) \Rightarrow -11 + 3x + 2x = 9$

$\Rightarrow 5x = 20 \Rightarrow \underline{x = 4}$

10. Find the maximum value of $2x + y$ on the feasible set given by the constraints:

$x + 10y \leq 50, \quad x + y \leq 5, \quad 10x + y \leq 50$

$x \geq 0, \quad y \geq 0.$



CORNER PTS	$2x+y$
(0,5)	5
(5,0)	10
(0,0)	0

ANSWER: 10

11. Let $A = \begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix}$. Then $A^{-1} = \begin{pmatrix} -1 & 1 \\ 4 & -3 \end{pmatrix}$. For a 2×2 matrix B, the following equation holds true:

$$BA^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Find the (2,1) entry of B. That is find the entry in the second row first column of B.

$$BA^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow (BA^{-1})A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}A \Rightarrow B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 3 & 1 \end{pmatrix}$$

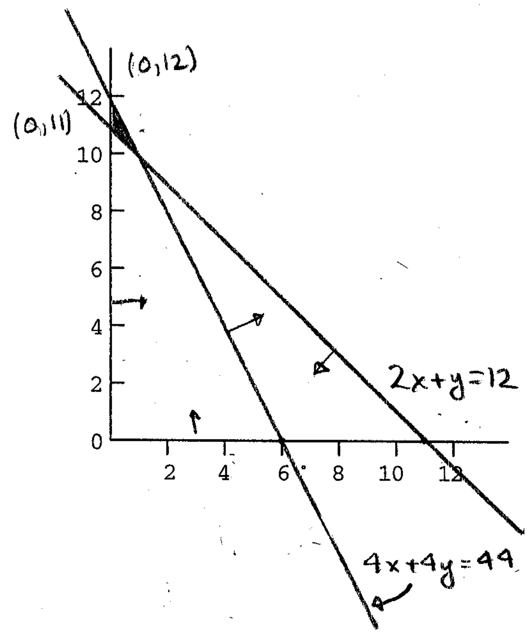
2nd ROW
1ST COLUMN

ANSWER: 3

12. Find the maximum value of $x + 2y$ on the feasible set given by the constraints:

$$2x+y \leq 12, \quad 4x+4y \geq 44, \quad x \geq 0, \quad y \geq 0.$$

You may find the rough sketch given to the right useful. Pay close attention to the direction of the inequalities given above.



$\begin{matrix} 2x+y=12 \\ 4x+4y=44 \end{matrix} \Rightarrow \begin{matrix} 2x+y=12 \\ 2x+2y=22 \end{matrix} \Rightarrow y=10$

$\Rightarrow x=1. \Rightarrow (1,10)$ is a corner pt.

CORNER PT.	$x+2y$
(0,11)	22
(0,12)	24
(1,10)	21

Answer: $\underline{\underline{24}}$

13. Given the system of equations

$$x - 4y + 5z = 9$$

$$2y - 6z = 12$$

$$3y + z = 8$$

Find z. $\begin{pmatrix} 1 & -4 & 5 & 9 \\ 0 & 2 & -6 & 12 \\ 0 & 3 & 1 & 8 \end{pmatrix} \xrightarrow[2R_1]{3R_2} \begin{pmatrix} 1 & -4 & 5 & 9 \\ 0 & 6 & -18 & 36 \\ 0 & 6 & 2 & 16 \end{pmatrix}$

$\xrightarrow{-R_2+R_3} \begin{pmatrix} 1 & -4 & 5 & 9 \\ 0 & 6 & -18 & 36 \\ 0 & 0 & 20 & -20 \end{pmatrix} \xrightarrow{1/20\, R_3} \begin{pmatrix} 1 & -4 & 5 & 9 \\ 0 & 6 & -18 & 36 \\ 0 & 0 & 1 & -1 \end{pmatrix} \Rightarrow 1 \cdot z = -1 \Rightarrow \underline{\underline{z = -1}}$

14. Find the $(1,2)$ entry in the matrix $2A - B$ where

$$A = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

$$2\begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

$$2 \cdot 3 - (-1) = \underline{\underline{7}}$$

15. An enterprising student plans to make and sell novelty T-shirts at the Little 500. A simpler style requires 1 ounce of ink and 20 minutes of labor for each shirt, while a more elaborate style requires 3 ounces of ink and 70 minutes to make each shirt. The student has 1/2 gallon of ink and can spend at most 20 hours making the T-shirts. The profit on the simpler style is \$3 per shirt, and the profit on the more elaborate shirt is \$5. Let x represent the number of simpler style shirts and y the number of more elaborate style shirts. The student wishes to maximize the profit made from this endeavor. Which of the following provides a complete formulation for the related linear programming problem. [Recall that there are 128 ounces in a gallon.]

Maximize $3x + 5y$ subject to

	$x+3y \le 64$	$20x+70y \le 1200$
(a) $3x + y \le 64$, $20x + 70y \le 1200$, $x \ge 0$, $y \ge 0$		
(b) $x + 3y \le 128$, $20x + 70y \le 1200$, $x \ge 0$, $y \ge 0$		X
(c) $x + 3y \le 128$, $20x + 70y \le 20$, $x \ge 0$, $y \ge 0$		
(d) $3x + y \le 64$, $70x + 20y \le 20$, $x \ge 0$, $y \ge 0$		
(e) $x + 3y \le 64$, $20x + 70y \le 1200$, $x \ge 0$, $y \ge 0$	X	X

$20x + 70y \le 20 \cdot 60 = 1200$

ANSWER: E

16. A feasible set for a linear programming problem is defined by

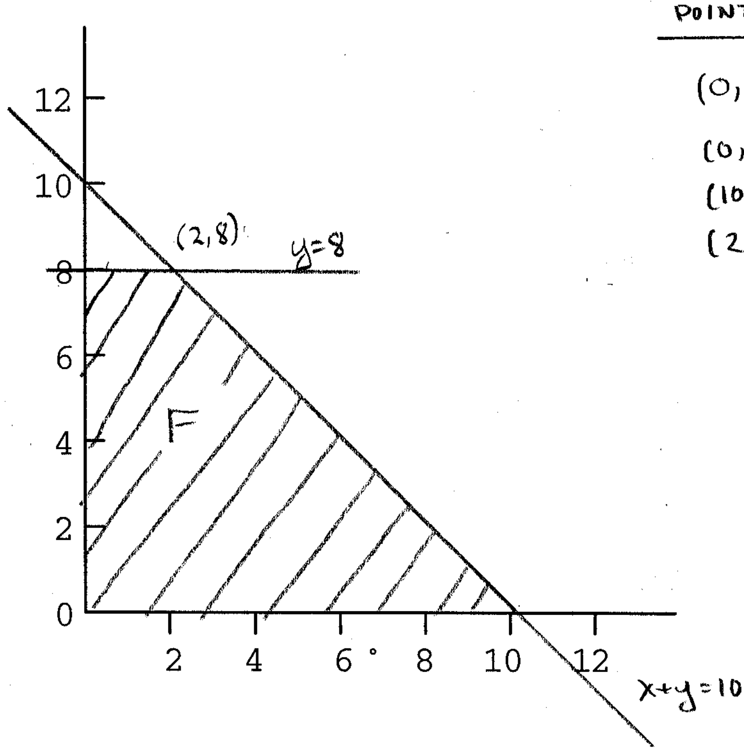
$$x \geq 0$$

$$y \geq 0$$

$$x + y \leq 10$$

$$y \leq 8$$

Find the maximum of $2x + 3y$ subject to these constraints. Pay careful attention to the direction of the inequalities given above.



CORNER POINTS	$2x+3y$
(0,8)	24
(0,0)	0
(10,0)	20
(2,8)	28

Answer: 28