1) Two lines with slopes m_1 and m_2 are perpendicular provided $m_1 = -\frac{1}{m_2}$. Let L_1 and L_2 be two lines given by

1): L_1 is the line running thru the points $(4, 0)$ and $(0, 6)$
2): L_2 is the line running thru the point $(0, 0)$ and perpendicular to L_1.

For some value of c the point $(6, c)$ lies on the line L_2. Find c.

a) -4 b) 9 c) -9

d) 0 e) 4 f) none of the above

L_1 has slope $m = \frac{6-0}{0-4} = -\frac{3}{2}$. Any line perpendicular to L_1 will have slope $= \frac{-1}{(-3/2)} = 2/3$. So the equation for L_2 will be of the form $y = 2/3\,x + b$. Now $(0,0)$ must satisfy this equation (by 2)); so $0 = 2/3 \cdot 0 + b$ $\Rightarrow b = 0$. The equation for L_2 is $y = 2/3\,x$. Setting $x = 6$ yields $y = 2/3 \cdot 6 = 4$. So $(6,4)$ lies on L_2.

Answer: 4

2) A line runs thru the points $(1,9)$ and $(6,-1)$. Another line runs thru the points $(2,7)$ and $(3,11)$. Let (x_0, y_0) be the point of intersection of these two lines. Find y_0.

a) 8 b) $\frac{4}{3}$ c) 16

d) 20 e) 7 f) none of the above

<u>Line I)</u>: Slope $= m_1 = \frac{9-(-1)}{1-6} = \frac{10}{-5} = -2$. So $y = -2x + b$.

Feed in $(1,9)$ to yield $9 = -2 \cdot 1 + b \Rightarrow b = 11$. Equation of line I) is $y = -2x + 11$.

<u>Line II)</u>: Slope $= m_2 = \frac{11-7}{3-2} = 4$, So $y = 4x + b$. Feed in $(2,7)$ to yield $7 = 4 \cdot 2 + b \Rightarrow b = -1 \Rightarrow y = 4x - 1$.

Now (x_0, y_0) must satisfy both eqs. So $-2x_0 + 11 = y_0 = 4x_0 - 1$

$\Rightarrow 12 = 6x_0 \Rightarrow 2 = x_0$. So $y_0 = 4 \cdot 2 - 1 = \underline{\underline{7}}$.

3) Let C be a constant such that the equation $4x + 6y = C$ describes a line passing thru the point $(3, 2)$. Find the slope of this line.

a) $-\frac{1}{3}$ b) -4 c) 4

d) $-\frac{2}{3}$ e) $-\frac{3}{2}$ f) none of the above

3) $4x + 6y = C \Rightarrow 6y = C - 4x \Rightarrow y = \frac{-4}{6}x + \frac{C}{6} = -\frac{2}{3}x + \frac{C}{6}.$

$\Rightarrow$ Slope of the line is $-2/3$.

5) Suppose

$$A = \begin{bmatrix} 7 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

Find x_1.

a) 3 b) 2 c) -2

d) 0 e) 1 f) none of the above

5) $\begin{pmatrix} 7 & -1 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7x_1 - 1x_2 \\ -4x_1 + 1x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

So
$$\begin{array}{l} 7x_1 - x_2 = 2 \\ -4x_1 + x_2 = 7 \\ \hline 3x_1 + 0 = 9 \end{array} \quad \text{ADD UP}$$

$\Rightarrow x_1 = 3. \Rightarrow x_2 = 19$

4) Consider the following matrix equation:

$$\begin{pmatrix} x & y & z \\ 2 & 1 & 1 \\ 5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 6 & 2 & 1 \end{pmatrix}.$$

Find x.

a) -2 b) 2 c) -1

d) 1 e) 0 f) none of the above

4) $$\underbrace{\begin{pmatrix} x & y & z \\ 2 & 1 & 1 \\ 5 & 1 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}}_{B} = \underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 6 & 2 & 1 \end{pmatrix}}_{C}$$

The (1,1) entry of AB is $\underbrace{(x\ y\ z)}_{1^{st}\text{ row of }A} \underbrace{\begin{pmatrix}1\\1\\0\end{pmatrix}}_{1^{st}\text{ column of }B} = x+y+0 = x+y$

I) This must equal the (1,1) entry of C, So $x+y=1$.

II) Likewise $(x\ y\ z)\underbrace{\begin{pmatrix}0\\1\\1\end{pmatrix}}_{2^{nd}\text{ column of }B} = y+z = 2$ ← (1,2) entry of C.

III) and $(x\ y\ z)\begin{pmatrix}0\\0\\1\end{pmatrix} = z = 3$. ← The (1,3) entry of C.

So $z=3 \Rightarrow$ (by II) $y=-1 \Rightarrow$ (by I) $\underline{\underline{x=2}}$.

5) Suppose

$$A = \begin{bmatrix} 7 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

Find x_1.

a) 3 b) 2 c) -2

d) 0 e) 1 f) none of the above

5) $\begin{pmatrix} 7 & -1 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7x_1 - 1x_2 \\ -4x_1 + 1x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

So $7x_1 - x_2 = 2$ ADD UP

$-4x_1 + x_2 = 7$

$3x_1 + 0 = 9 \Rightarrow x_1 = 3. \Rightarrow x_2 = 19$

6) Solve the following system to find x.

$$\begin{aligned} 2x + y + 2z &= 2 \\ x + y + z &= 4 \\ 2x + 0y + z &= 6 \end{aligned}$$

a) 10 b) -8 c) 6

d) 0 e) 8 f) none of the above

The associated augmented matrix is:

$$\begin{pmatrix} 2 & 1 & 2 & 2 \\ 1 & 1 & 1 & 4 \\ 2 & 0 & 1 & 6 \end{pmatrix}. \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 1 & 1 & 4 \\ 2 & 1 & 2 & 2 \\ 2 & 0 & 1 & 6 \end{pmatrix} \xrightarrow{-2R_1+R_2}$$

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & -1 & 0 & -6 \\ 2 & 0 & 1 & 6 \end{pmatrix} \xrightarrow{-2R_1+R_3} \begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & -1 & 0 & -6 \\ 0 & -2 & -1 & -2 \end{pmatrix} \xrightarrow{-2R_2+R_3}$$

$$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & -1 & 0 & -6 \\ 0 & 0 & -1 & 10 \end{pmatrix} \Rightarrow \left.\begin{aligned} x + y + z &= 4 \\ y &= 6 \\ z &= -10 \end{aligned}\right\} \quad x + 6 + (-10) = 4$$

$$\underline{\underline{x = 8}}.$$

7) A system of 3 equations in 4 unknowns has an augmented matrix given by

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\begin{pmatrix} 0 & 1 & 1 & 1 & 5 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & 3 \end{pmatrix}.$$

Which of the following statements about this system are correct?

a) The system has no solutions.
b) The system has exactly one solution.
c) The system has an infinite number of solutions with exactly one arbitrary parameter (i.e. one free variable).
d) The system has an infinite number of solutions with exactly two arbitrary parameters (i.e. two free variables).
e) The system has an infinite number of solutions with exactly three arbitrary parameters (i.e. three free variables).
f) The system has an infinite number of solutions with exactly four arbitrary parameters (i.e. four free variables).

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 5 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & 3 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 5 \\ 1 & 1 & 1 & 1 & 3 \end{pmatrix} \xrightarrow{-R_1+R_3}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{-R_2+R_3} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & X_3 & X_4 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 5 \\ 0 & 0 & -1 & 0 & -4 \end{pmatrix} \end{array}$$

This is not in reduced form, but it is obvious that there will be 3 pivots, and hence one free variable. Answer: c).

8) A system of 3 equations in 4 unknowns has an augmented matrix given by

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\begin{pmatrix} 0 & 1 & 1 & 1 & 5 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 & 8 \end{pmatrix}.$$

Which of the following statements about this system are correct?

a) The system has no solutions.
b) The system has exactly one solution.
c) The system has an infinite number of solutions with exactly one arbitrary parameter (i.e. one free variable).
d) The system has an infinite number of solutions with exactly two arbitrary parameters (i.e. two free variables).
e) The system has an infinite number of solutions with exactly three arbitrary parameters (i.e. three free variables).
f) The system has an infinite number of solutions with exactly four arbitrary parameters (i.e. four free variables).

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 5 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 & 3 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 5 \\ 1 & 1 & 2 & 1 & 8 \end{pmatrix}$$

You don't really need to do this, but it is a reasonable first step (as in Prob. 8)

$$\xrightarrow{-R_1+R_2} \begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1 & 3 \end{pmatrix} \xrightarrow{-R_2+R_3} \begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix} \leftarrow$$

The last line implies $0x_1 + 0x_2 + 0x_3 + 0x_4 = -2$. There is no solution to this equation. Answer: a).

9) A system of 3 equations in 5 unknowns has an augmented matrix given by

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{array}$$
$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 6 \\ 1 & 2 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}.$$

IF the corresponding system of equations is solved for x_2 in term of the free variables, and IF all of the free variables are given (i.e. set to) the value 10, then the value for x_2 would be:

a) -26

b) -24

c) -22

d) -16

e) 2

f) none of the above

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 6 \\ 1 & 2 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow{-2R_2+R_1} \begin{pmatrix} 1 & 0 & -1 & -2 & -2 & -9 \\ 0 & 1 & 1 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{-R_3+R_2}$$

$$\begin{pmatrix} 1 & 0 & -1 & -2 & -2 & -9 \\ 0 & 1 & 1 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$ Any further row ops. will not affect R_2 (which has x_2 as a pivot).

So $1 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 = 4 \Rightarrow x_2 = 4 - x_3 - x_4$. Setting

$x_3 = x_4 = 10$ yields $x_2 = 4 - 10 - 10 = \underline{\underline{-16}}$.

10) Let

$$A = \begin{pmatrix} -5 & -2 \\ 3 & 1 \end{pmatrix}$$

Find the entry in the **second row** and **second column** of A^{-1}.

a) 0 b) -5 c) -3

d) 2 e) 5 f) none of the above

$$\left(\begin{array}{cc|cc} -5 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) \xrightarrow[5R_2]{3R_1} \begin{pmatrix} -15 & -6 & 3 & 0 \\ 15 & 5 & 0 & 5 \end{pmatrix} \xrightarrow{R_1+R_2}$$

$$\begin{pmatrix} -15 & -6 & 3 & 0 \\ 0 & -1 & 3 & 5 \end{pmatrix} \xrightarrow{-6R_2+R_1} \begin{pmatrix} -15 & 0 & -15 & -30 \\ 0 & -1 & 3 & 5 \end{pmatrix}$$

$$\xrightarrow[-1R_2]{-1/15 R_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -3 & -5 \end{pmatrix}$$

Answer: -5

Check: $\begin{pmatrix} -5 & -2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -3 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

11) A toy manufacturer sells clothing outfits to Mattel for Barbie dolls. Clothing ensemble A consists of 2 dresses and 3 blouses. Clothing ensemble B consists of 3 dresses and 5 blouses. All of the blouses are all made of one material. All of the dresses are made of one material (<u>different</u> from the material used for the blouses). The manufacturer has enough material to make 36,000 dresses and 55,000 blouses. The manufacturer uses up ALL of these materials making up A ensembles and B ensembles. How many B ensembles were made?

a) 6,000 b) 8,000 c) 12,000

d) 14,000 e) 2,000 f) none of the above

15) Let $x = \#$ A ensembles made

$y = \#$ B ensembles made

$\#$ dresses made $= 2x + 3y = 36{,}000$

$\#$ blouses made $= 3x + 5y = 55{,}000$

$\times 2$ $\times 3$

$\Rightarrow$ $6x + 9y = 108{,}000$

$6x + 10y = 110{,}000$

$y = 2000 \Rightarrow x = 15{,}000$

Answer: <u>2,000</u>

12) A society produces two goods - frisbees and nerf footballs. *Strange* as it may seem, the production of these goods is governed by a Leontief model for which:

a) The production of one case of frisbees requires .9 cases of frisbees and .4 cases of nerf footballs.

b) The production of one case of nerf footballs requires .1 cases of frisbees and .4 cases of nerf footballs.

There is an external demand for 10 cases of frisbees and 20 cases of nerf footballs. To meet this external demand will require the production of a certain number of cases of frisbees and cases of nerf footballs (i.e. a certain production schedule will have to be met). Find the number of cases of nerf footballs that will have to be produced.

a) 4	b) 300	c) 400
d) 700	e) 150	f) none of the above

$$A = \begin{pmatrix} .9 & .1 \\ .4 & .4 \end{pmatrix} \qquad I - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} .9 & .1 \\ .4 & .4 \end{pmatrix} = \begin{pmatrix} .1 & -.1 \\ -.4 & .6 \end{pmatrix}$$

The problem is equivalent to solving $(I - A)x = d \iff$

$\begin{pmatrix} .1 & -.1 \\ -.4 & .6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 20 \end{pmatrix}$. Form the associated augmented matrix to obtain

$$\left(\begin{array}{cc|c} .1 & -.1 & 10 \\ -.4 & .6 & 20 \end{array}\right) \xrightarrow{4R_1 + R_2} \left(\begin{array}{cc|c} .1 & -.1 & 10 \\ 0 & .2 & 60 \end{array}\right) \xrightarrow[5R_2]{10R_1}$$

$$\begin{pmatrix} 1 & -1 & 100 \\ 0 & 1 & 300 \end{pmatrix} \xrightarrow{R_2 + R_1} \begin{pmatrix} 1 & 0 & 400 \\ 0 & 1 & 300 \end{pmatrix}$$

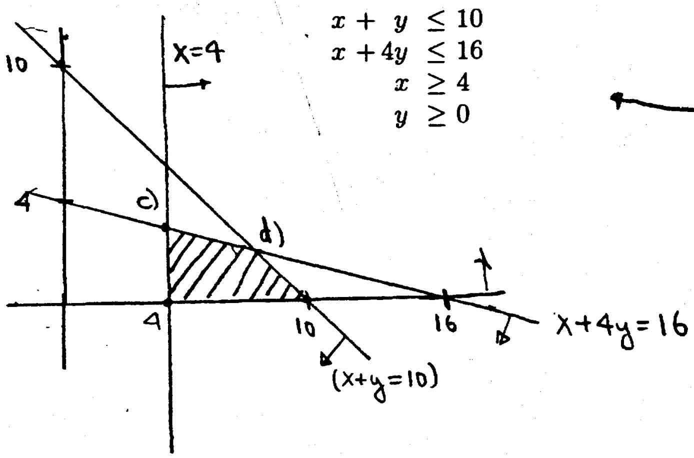
Answer: 300

Note: This problem and the problem immediately following this have the **same** constraints. Sketch the feasible set carefully since you will need it for two problems.

13) Find the maximum of $2x + 5y$ subject to the constraints

$$x + y \leq 10$$
$$x + 4y \leq 16$$
$$x \geq 4$$
$$y \geq 0$$

Problem 14: Maximize $x+5y$ subject to these same constraints.



The corner points are (4,0), (10,0) and c) and d).

c) is located at "(x=4) ∩ (x+4y=16)" $\Rightarrow$ $4+4y=16 \Rightarrow y=3$;

so, c) is located at (4,3)

d) is located at "(x+y=10) ∩ (x+4y=16)".

$$\begin{array}{r} x+4y=16 \\ -\,(x+y=10) \\ \hline 3y=6 \end{array}$$

$3y=6 \Rightarrow y=2 \Rightarrow x=8$. So d) is at (8,2)

Corner Pts.	2x+5y =	x+5y
(4,0)	$2\cdot4+5\cdot0=8$	4
(4,3)	$2\cdot4+5\cdot3=23$	19 ← MAX
(8,2)	MAX → $2\cdot8+5\cdot2=26$	18
(10,0)	$2\cdot10+5\cdot0=20$	10

Answer to 14)

15) Consider the following matrix equation:

$$\underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 0 & 2 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 5 & -2 & 1 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{pmatrix}}_{A^{-1}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{I}.$$

Suppose that C is a 3×3 matrix such that

$$\star \qquad \underbrace{\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 0 & 2 & 1 \end{pmatrix}}_{A} \; C = \underbrace{\begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}}_{D}.$$

Find the $(2,3)$ entry of C (i.e. find the entry in **row 2** and **column 3** of C).

a) -2 b) -16 c) 7

d) 18 e) -4 f) none of the above

The given equation $\star$ is $AC = D$. You know A and D. The problem is to find C. Multiply this equation on both sides on the <u>left</u> by A^{-1} to get

$$A^{-1}(AC) = A^{-1}D.$$

But $A^{-1}(AC) = (A^{-1}A)C = IC = C$. So

$$C = A^{-1}D \quad \text{or}$$

Suppose you were asked to solve $CA = D$ where you know A and D; multiply by A^{-1} on the right on both sides

$$C = CAA^{-1} = DA^{-1}$$

$$C = A^{-1}D = \underbrace{\begin{pmatrix} 5 & -2 & 1 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{pmatrix}}_{A^{-1}} \underbrace{\begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}}_{D} = \underbrace{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & * \\ \cdot & \cdot & \cdot \end{pmatrix}}_{C}$$

3×3

FIND THIS ENTRY – THE (2,3) ENTRY

$$\text{ROW 2} \rightarrow \underbrace{\begin{pmatrix} 5 & -2 & 1 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{pmatrix}}_{A^{-1}} \underbrace{\begin{pmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{pmatrix}}_{D} \longrightarrow -2 \cdot 7 + 1 \cdot 10 + 0 \cdot 13 = \underline{\underline{-4}}$$

COLUMN 3

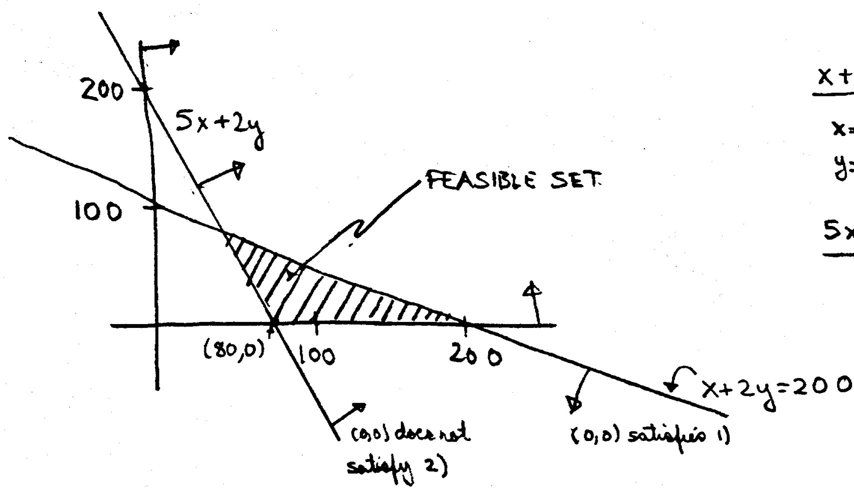
16) Consider the feasible set described by the following inequalites

$$\begin{array}{lrcl} 1) & x + 2y & \leq & 200 \\ 2) & 5x + 2y & \geq & 400 \\ 3) & x & \geq & 0 \\ 4) & y & \geq & 0 \end{array}$$

Which **three** of the points listed below are the corner points of this set? List them on the space provided on the answer sheet. (For example a typical answer on your answer sheet might look like a,d,e. Note that you have to get the answer exactly correct to get credit.) Pay close attention to the inequalities, some are $\geq$ and some are $\leq$.

a) (50,75) b) (0,100) c) (0,200)

d) (80,0) e) (200,0) f) (0,0)



$\underline{x+2y=200}$ PTS. ON LINE

$x=0 \Rightarrow y=100$ (0,100)

$y=0 \Rightarrow x=200$ (200,0)

$\underline{5x+2y=400}$

$x=0 \Rightarrow y=200$ (0,200)

$y=0 \Rightarrow x=80$ (80,0)

$$\begin{array}{l} x+2y=200 \\ -\underline{(5x+2y=400)} \\ -4x+0=-200 \Rightarrow \underline{\underline{x=50}} \Rightarrow 50+2y=200 \Rightarrow \underline{\underline{y=75}} \end{array}$$

So $\underline{(50,75)}$ is a corner pt., and so are $\underline{(80,0)}$ and $\underline{(200,0)}$.

Set-up for problems 17 and 18: A sailplane manufacturer makes two models of sailplanes - the Stratus and the Cumulus. To build one Stratus sailplane requires 72 sheets of carbon fiber material (and other associated materials) for making structures and surfaces, 800 hours of labor, and one (1) hardware package (control fittings, hinges, linkages, cables, etc.). To build one Cumulus sailplane requires 36 sheets of carbon fiber material (and associated materials), 500 hours of labor, and one (1) hardware package (the same package as used for the Stratus). The manufacturer makes \$20,000 profit on each Stratus plane and \$10,000 on each Cumulus. He has available 10,800 carbon fiber sheets (and associated materials), 130,000 hours of labor and 200 hardware packages for the next the production run. How many planes of each model should be built in order to maximize profits?

NOTICE

Let,

p = # of hardware packages used to build all Stratus sailplanes
q = # of hardware packages used to build all Cumulus sailplanes
r = # of hours of labor needed to build all Stratus sailplanes
s = # of hours of labor needed to build all Cumulus sailplanes
t = # of carbon fiber sheets used to build all Stratus sailplanes
u = # of carbon fiber sheets used to build all Cumulus sailplanes
a = # of Stratus sailplanes built
b = # of Cumulus sailplanes built ← You have control over this
c = the profit per Stratus sailplane made
d = the profit per Cumulus sailplane made.

---

17) For the linear programming problem corresponding to this set-up, what is the objective function? Be careful to select the variables correctly from the list above.

Answer $20{,}000a + 10{,}000b$

You are being asked how many Stratus and how many Cumulus to build. This is what you have control over, so let these quantities be the variables.

Profit/Cumulus # Cumulus

$$\underbrace{20{,}000a}_{\text{TOTAL PROFIT FROM STRATUS}} + \underbrace{10{,}000b}_{\text{TOTAL PROFIT FROM CUMULUS}} = \text{TOTAL PROFIT}$$

Profit/Stratus # Stratus

18) For the linear programming problem corresponding to this set-up, list below the constraint equations. There are **more** lines than constraint equations: 2 points for every correct equation listed, -1 point for every equation listed that is not a constraint equation. Note also that just because an inequality may hold true, does not necessarily mean that it is a constraint equation (e.g. $d \leq c$). List only those equations which are truly constraint equations as determined by the set-up above. Be careful to select the variables correctly from the list above.

$a \geq 0$

$b \geq 0$

$72a + 36b \leq 10{,}800$

$800a + 500b \leq 130{,}000$

$a + b \leq 200$

The total number of carbon fiber sheets used is

used/Cumulus

$$72a + 36b$$

Cumulus

of Strato

used/Strato

There are 10,800 sheets available $\Rightarrow$ $72a + 36b \leq 10{,}800$.

The constraint involving labor is

$$800a + 500b \leq 130{,}000.$$

The total # hardware packages used is $a + b$ (one for each sailplane made). So

$$a + b \leq 200.$$