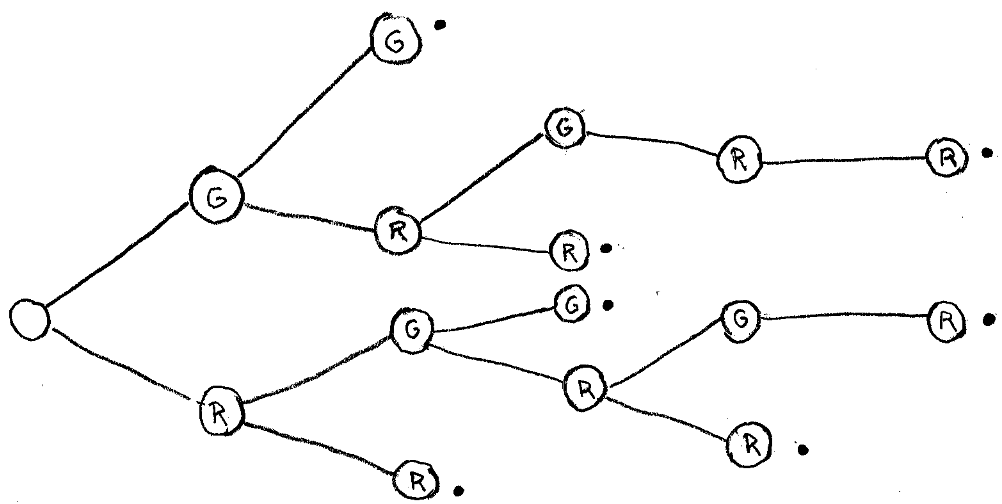
1) A hat contains 3 red slips of paper and 2 green slips of paper. One slip after another is drawn from the hat, without replacement. The colors of the drawn slips as well as the order in which they were drawn is recorded. The process is terminated whenever the same color is drawn twice in a row or there are no more slips left in the hat. What is the size of the corresponding sample space.? Hint: Draw a tree.

Example: One outcome would be RGRGR (red, then green, then red, then green, then red), another is GG.

TREE:



7 dots

Answer: 7

2) Adam, Barb, Candy, Doug, and Earl go to the movie theater and sit in a row with exactly 5 seats. How many different ways can they arrange themselves?

Seat $\overline{\ \ }$ 1 $\overline{\ \ }$ 2 $\overline{\ \ }$ 3 $\overline{\ \ }$ 4 $\overline{\ \ }$ 5

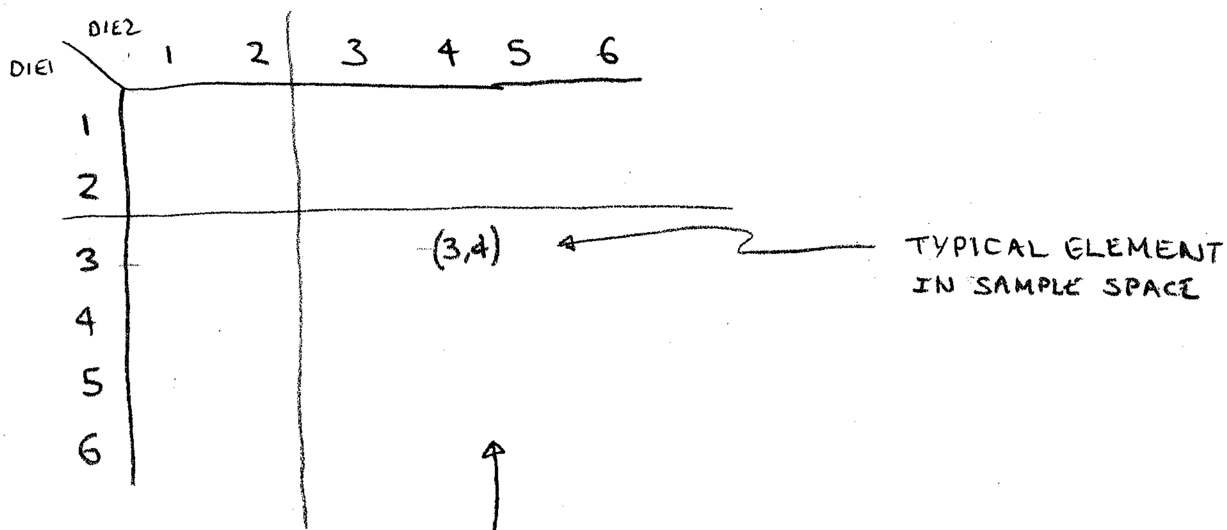
5 choices for the 1st seat, 4 for the second, 3 for the 3rd . . .

Answer: $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Or $P(5,5) = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 120$

3) You roll two fair dice and look at the result. What is the probability that you do not see a 1 or a 2 on either die?

Example: Rolling a 4 on the first die and a 5 on second - neither die came up with a 1 or 2.



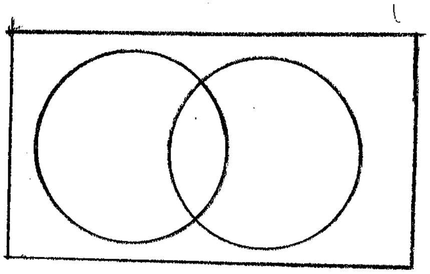
Only those elements in here don't have a 1 or 2 in them. This is a 4×4 region.

\# outcomes with no 1's or 2's

Answer: $\dfrac{4 \times 4}{36} = \dfrac{16}{36}$

\# possible outcomes (all <u>equally likely</u>)

4) Find $n(A \cap B)$, given that A and B are subsets of U with $n(U) = 100$, $n(A') = 77$, $n(B) = 15$, and $n(A \cup B) = 31$.



$n(A') = 77 \Rightarrow n(A) = n(U) - n(A')$

$= 100 - 77 = 23$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$31 = 23 + 15 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 23 + 15 - 31 = \underline{\underline{7}}$

5) You are casting a play. There is one female role to be cast: Old Mother Hubbard. And there are three male roles to be cast: the Butcher, the Baker, and the Candlestick Maker. 3 women and 4 men try out. How many ways can you cast the play?

$C(3,1)$

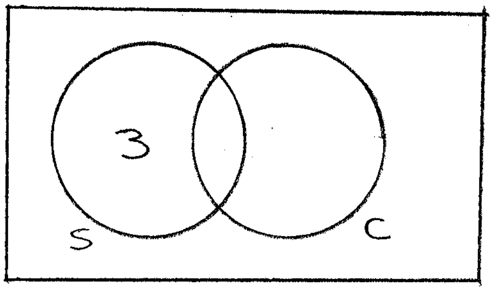
ways to choose one of the 3 women

$4 \cdot 3 \cdot 2$

$P(4,3)$ = # ways to select 3 of the men, keeping track of order.

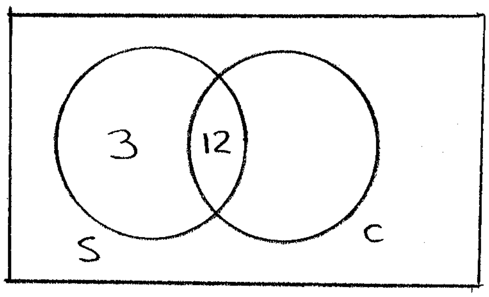
Answer: $3 \cdot 4 \cdot 3 \cdot 2 = \underline{\underline{72}}$

6) Of a group of 100 people, 15 smoke, 42 drink coffee, and 3 smoke but don't drink coffee. How many drink coffee but don't smoke?

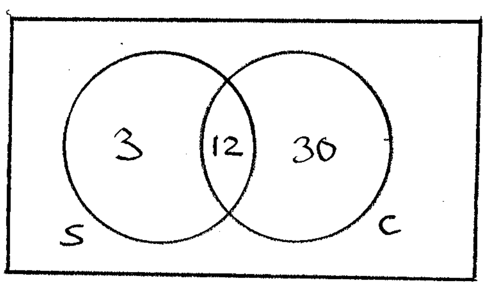


$n(S) = 15$

$n(C) = 42$



Use $n(S)=15$



Use $n(S)=42$

Answer: 30

7) How many 5 digit numbers are there that consist of only 4's and 8's?

Examples: 44888, 44444, 84848, 84444.

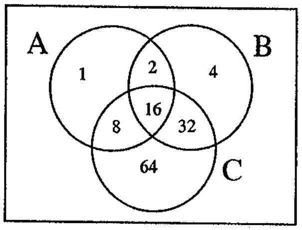
2 choices 2 choices

_ _ _ _ _ 2 choices

2 choices 2 choices

$2 \times 2 \times 2 \times 2 \times 2 = 32$

8) Let A, B, C be subsets of a universal set U where $n(U) = 412$. Shown below is a Venn diagram for the sets A, B, C (which has been labelled with the number of elements in its various subsets). How many elements are in the set $(A' \cup B' \cup C')$?



$(A' \cup B' \cup C') = (A \cap B \cap C)'$

$n(A \cap B \cap C) = 16$

$\Rightarrow \quad n((A \cap B \cap C)') = n(U) - n(A \cap B \cap C)$

$= 412 - 16 = \underline{\underline{396}}$

9) Suppose Ω is a universal set with $n(\Omega) = 100$, and suppose A, B, and C are subsets of Ω with:

$n(A) = n(B) = n(C) = 50$

$n(A \cap B) = n(B \cap C) = n(A \cap C) = 30$

$n((A \cup B \cup C)') = 22$

$n(A \cup B \cup C) = \underset{n(\Omega)}{100} - 22 = 78$

What is $n(A \cap B \cap C)$?

$$n(A \cup B \cup C) = \underbrace{n(A) + n(B) + n(C)}_{150} - \underbrace{n(A \cap B) - n(B \cap C) - n(A \cap C)}_{90} + n(A \cap B \cap C)$$

$$78 = 150 - 90 + n(A \cap B \cap C)$$

$$78 = 60 + n(A \cap B \cap C)$$

$$\Rightarrow \quad n(A \cap B \cap C) = 18$$

10) A hat contains 5 red slips of paper and 7 green slips of paper. Two slips are drawn out of the hat, at random, one after the other, and without replacement. What is the probability that both slips are red?

ways to draw out 2 red slips.

$$\frac{C(5,2)}{C(12,2)}$$

ways to draw out any 2 slips

$$= \frac{5\cdot4/2}{12\cdot11/2} = \frac{10}{66}$$

11) How many 3 letter words can be formed using the letters AABBCCDDEEFF?

Example: Here are some 3 letter words that can be formed: ACA, FDC, EBC.

Hint: How many 3 letter words are possible using the letters AAABBBCCCDDDEEEFFF (i.e. there are no "restrictions")?

Without restrictions

6 choices (A, B, C, D, E, or F)

__ __ __

6 choices 6 choices.

$6 \cdot 6 \cdot 6 = 216$

But using just AABBCCDDEEFF, no word with all 3 letters the same can be formed. There are 6 such words AAA, BBB, ..., FFF.

Answer: $216 - 6 = 210$

12) Ima Quack has 6 patients in the waiting room, 2 men and 4 women. One patient is selected at random to see Dr. Quack and then another (at random). What is the probability that Dr. Quack sees a male patient and then a female patient?

ways to select 1M, then 1F.

ways to select 1 male patient

ways to select 1 female patient

$$\frac{2 \cdot 4}{6 \cdot 5}$$

ways to select 2 patients keeping track of order

Answer: $\frac{8}{30} = 4/15$

13) Five boys and two girls are seated in 7 seats numbered 1 through 7. In how many ways can this be done so that the 2 girls are seated in seats 1 and 2?

There are 2 ways to seat the girls ($\underline{G_1}\ \underline{G_2}$ or $\underline{G_2}\ \underline{G_1}$)
For each one of these ways, there are 5! ways to seat the boys. (see problem 2 of these solutions).

Answer: $2! \cdot 5! = 2 \cdot 120 = 240$

14) A six-sided die is weighted so that rolling a 1, 2, 3, and 4 are equally likely and rolling a 5 is 2 times as likely as rolling a 4 and rolling a 6 is 2 times as likely as rolling a 4. What is the probability of rolling a 2?

$$w(1) = w(2) = w(3) = w(4)$$

$$w(5) = w(6) = 2w(4)$$

$$w(1) + w(2) + w(3) + w(4) + w(5) + w(6) = 1$$

$$\Rightarrow$$

$$w(4) + w(4) + w(4) + w(4) + 2w(4) + 2w(4) = 1$$

$$\Rightarrow \quad 8w(4) = 1 \Rightarrow w(4) = 1/8 \Rightarrow \underline{\underline{w(2) = 1/8}}$$

15) You own 3 cars. Each is to be painted either red, or yellow, or black, or white. In how many ways can this be done in such a way that exactly 2 or the cars are the same color?

Examples: One way is to paint car 1 black, car 2 white, car 3 black. Another way is to paint car 1 red, car 2 red, and car 3 black.

Method 1: Select 2 cars to be the same color. There are $C(3,2) = 3$ ways to do this. Then select a color for these 2 cars. There are 4 choices. Then select a color for the remaining car. There are 3 colors left over - 3 choices.

Answer: $3 \cdot 4 \cdot 3 = 36$

Method 2: There are $4 \cdot 4 \cdot 4 = 64$ ways to paint the cars any color. Of these 64 ways, there are ① $4 \cdot 3 \cdot 2 = 24$ ways to paint each car a different color. And there are ② 4 ways to paint them all the same color. The opposite of ① and ② is to paint exactly 2 cars the same color.

Answer: $64 - (\underset{①}{24} + \underset{②}{4}) = \underline{\underline{36}}$