

1. A box contains 2 nickels and 3 quarters. Two coins are selected simultaneously and at random, and a random variable  $X$  is defined as the total value of the two coins selected. Find the probability  $\Pr[X=30]$ .

- A)  $3/5$   
 B)  $1/2$   
 C)  $3/10$   
 D)  $9/20$   
 E) none of the others

$$\Pr[X=30] = \Pr(N+Q) = \frac{2 \cdot 3}{C(5,2)} = \frac{6}{10} = \frac{3}{5}$$

2. An unfair die has the property that when rolled each of the odd numbers is equally likely to land uppermost, each of the even numbers is equally likely to land uppermost, and each odd number is twice as likely to land up as an even number. The die is rolled and the result is noted. Find the probability that the result is in the event  $\{1,3,6\}$ .

- A)  $4/9$   
 B)  $1/3$   
 C)  $5/9$   
 D)  $2/3$   
 E) none of the others

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 1$$

$$w_1 = w_3 = w_5 \quad w_2 = w_4 = w_6$$

$$3w_1 + 3w_2 = 1$$

$$w_1 = 2w_2$$

$$3(2w_2) + 3w_2 = 1$$

$$\Rightarrow w_2 = 1/9 \Rightarrow w_1 = 2/9$$

$$\{1, 3, 6\}$$

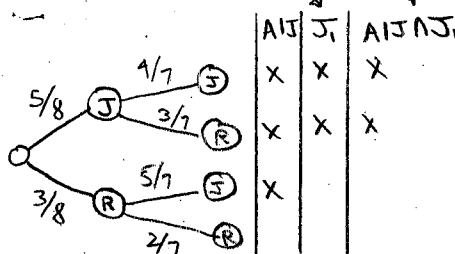
$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ 2/9 & 2/9 & 1/9 \end{matrix}$$

$$2/9 + 2/9 + 1/9 = 5/9$$

3. There are 8 CDs in a rack; 5 are jazz and 3 are classic rock. Two CDs are selected at random, one after the other without replacement, and the type of each is noted. Find the probability that the first is jazz given that at least one is jazz.

- A)  $5/7$   
 B)  $3/7$   
 C)  $5/8$   
 D)  $7/10$   
 E) none of the others

NOTE:  $J_1 = A1J \cup J_1$



$$\Pr(J_1 | A1J) = \frac{\Pr(J_1 \cap A1J)}{\Pr(A1J)}$$

$$= \frac{5/8}{1 - 3/8 \cdot 2/7} = \frac{35}{50}$$

4. An unfair coin with  $\Pr[H] = .6$  is flipped 3 times and the result of each toss is noted. What is the probability that there are at least two heads?

- A)  $(.6)^3 + 3(.6)^2(.4)$   
 B)  $(.4)^3 + 3(.4)^2(.6)$   
 C)  $3(.6)^2(.4)$   
 D)  $(.6)^2(.4)$   
 E) none of the others

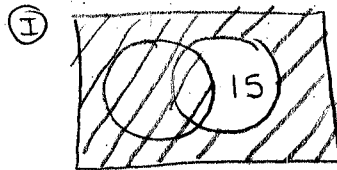
$$\Pr(A2H) = \Pr(2H) + \Pr(3H)$$

$$= C(3,2)(.6)^2(.4) + C(3,3)(.6)^3$$

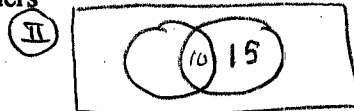
$$= 3(.6)^2(.4) + (.6)^3$$

5. Let A and B be subsets of a universal set U:  $n(U) = 60$ ,  $n(B) = 25$ , and  $n(A \cup B) = 45$ . Find  $n(A \cap B)$ .

- A) 10  
 B) 20  
 C) 30  
 D) 15  
 E) none of the others



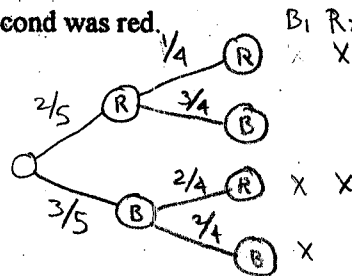
$A \cup B$        $(A \cup B)' = A' \cap B'$   
 $n(A \cap B) = 60 - 45 = 15$



$n(B) = 25$

6. A box contains 2 red balls and 3 blue balls. Two balls are selected at random, one after another without replacement, and the color of each is noted. Find the probability that the first was blue given that the second was red.

- A) 3/4  
 B) 1/4  
 C) 3/5  
 D) 2/5  
 E) none of the others



$B_1, R_2$        $X$   
 $Pr(B_1 | R_2)$   
 $= \frac{Pr(B_1 \cap R_2)}{Pr(R_2)}$   
 $= \frac{3/5 \cdot 2/4}{3/5 \cdot 2/4 + 2/5 \cdot 1/4} = \frac{6}{6+2}$

7. Suppose the free throws of a basketball player can be viewed as Bernoulli trials all with success probability  $p = .4$ . If she takes four shots in a game, and if each successful shot is worth one point, find the expected number of points scored.

- A) 1.0  
 B) 2.0  
 C) 1.6  
 D) 2.4  
 E) none of the others

$4 \times .4 = 1.6$   
 $n \quad p$

8. A fair coin is flipped until there are two consecutive tails or a total of four flips, and the result of each flip is noted. The event E consists of those outcomes with exactly 2 tails. Find  $Pr[E]$ . Flip coin times. 16 outcomes.

- A) 3/8  
 B) 5/8  
 C) 1/2  
 D) 1/4  
 E) none of the others

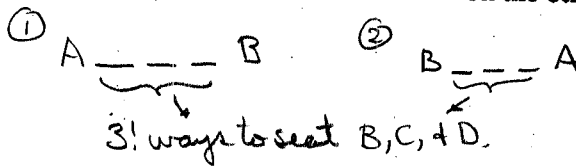
$C(4,2) = 6$  have exactly 2 T's  
 $C(4,3) = 4$  have exactly 3 T's  
 $C(4,4) = 1$  have exactly 4 T's

$11 - 1 = 10$        $10/16$

(TTTH) TERMINATES  
 (TTHT) TERMINATES  
 (THTT) ← 3 T's  
 (HTTT) TERMINATES  
 (HTTT) SHOWS UP AS 2 T AFTER TERMINATION

9. Amy, Bob, Carlos, Debra, and Erin have tickets for five adjacent seats at a concert. In how many ways can they be seated if Amy is on one end and Bob is on the other end?

- A) 24  
 B) 12  
 C) 30  
 D) 6  
 E) none of the others

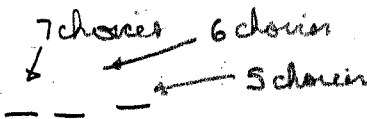


Answer:  $2 \cdot 3! = 12$

10. A product code is to be formed with three distinct letters from the set {B, H, K, Q, T, V, Z}. How many different product codes are there? Note: BHK is a different product code than KBH

- A) 7!  
 B)  $7!/3!$   
 C) 3!  
 D)  $7!/4!$   
 E) none of the others

$$P(7,3) = 7 \cdot 6 \cdot 5 = \frac{7!}{4!}$$



11. Suppose that for Joe the events "late for class" and "prepared for class" are independent events with  $\Pr[\text{late for class}] = .2$  and  $\Pr[\text{prepared for class}] = .6$ . Find the probability that Joe is late for class and is not prepared.

- A) .08  
 B) .8  
 C)  $1/3$   
 D) .12  
 E) none of the others



$$\Pr(L \cap P') = \Pr(L) \cdot \Pr(P') = .2 \cdot .4 = .08$$

↑  
1 - .6

12. A sample space has 5 outcomes with weights  $w_1, w_2, w_3, w_4,$  and  $w_5$ . If  $w_1 = 2w_2$ ,  $w_2 = w_3 = w_4$ , and  $w_5 = 3w_2$ , find  $w_4$ .

- A)  $1/5$   
 B)  $1/3$   
 C)  $1/8$   
 D)  $1/4$   
 E) none of the others

$$w_1 + w_2 + w_3 + w_4 + w_5 = 1$$

$$\Rightarrow 2w_2 + w_2 + w_2 + w_2 + 3w_2 = 1$$

$$\Rightarrow 8w_2 = 1 \Rightarrow w_2 = 1/8 \Rightarrow w_4 = 1/8$$

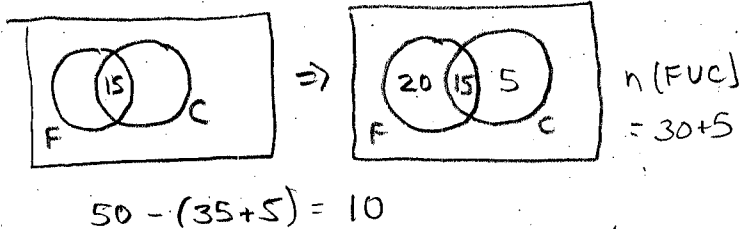
13. Three students are selected simultaneously and at random from a group consisting of 2 freshmen, 1 sophomore, and 3 juniors. Find the probability that the 3 students are from 3 different classes.

- A)  $1/20$   
 B)  $1/3$   
 C)  $3/10$   
 D)  $11/15$   
 E) none of the others

$$\frac{2 \cdot 1 \cdot 3}{C(6,3)} = \frac{6}{6 \cdot 5 \cdot 4 / 3 \cdot 2 \cdot 1} = \frac{6}{5 \cdot 4} = \frac{6}{20}$$

14. Fifty students are asked about their plans for the weekend. Thirty-five plan to go to a football game, twenty plan to attend a concert, and fifteen plan to do both. How many plan to do neither?

- A) 15  
 B) 20  
 C) 30  
 D) 10  
 E) none of the others



15. Let  $U = \{u, v, w, x, y, z, 2, 3\}$ ,  $E = \{2, 3, w, z\}$ ,  $F = \{2, 3, u, y, z\}$ , and  $G = \{1, 2, 3, w, y\}$ . Find  $(E \cup F) \cap G'$ .

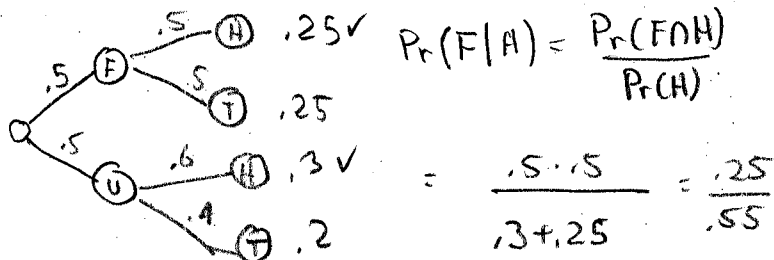
- A)  $\{2, 3, w, y\}$   
 B)  $\{u, v, x, z\}$   
 C)  $\{u, z\}$   
 D)  $\{2, u, z\}$   
 E) none of the others

$G' \Rightarrow$  Cross out elements in  $G$ .

$$(E \cup F) \cap G' = \{2, u, z\}$$

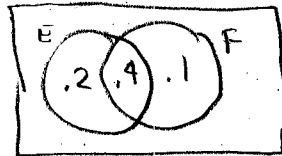
16. There are two coins, coin 1 has  $\Pr[H] = .5$  and coin 2 has  $\Pr[H] = .6$ . A coin is selected at random and flipped. If the result is a H, what is the probability coin 1 was selected?

- A)  $5/11$   
 B)  $3/5$   
 C)  $1/2$   
 D)  $6/11$   
 E) none of the others



17. Let E and F be events in a sample space with  $\Pr[E] = .6$ ,  $\Pr[E \cap F] = .4$  and  $\Pr[F] = .5$ . Find  $\Pr[E|F]$ .

- A)  $2/5$   
 B)  $4/5$   
 C)  $5/6$   
 D)  $2/3$   
 E) none of the others



$$\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)}$$

$$= \frac{.4}{.5}$$

18. A random variable X has the probability density function shown below. Find the expected value of X.

value of X	probability	(value of X) · Pr
-1	.2	-.2
1	.4	.4
4	.3	1.2
5	.1	+.5
		1.9

- A) 1.0  
 B) 2.4  
 C) 2.1  
 D) 1.9  
 E) none of the others

19. There are 3 male roles in a play and there are 5 males auditioning for these roles, including Alex. Also, there are 2 female roles in the same play and there are 4 females auditioning, including Zelda. If each person is equally likely to be assigned a role, find the probability that exactly one of Alex or Zelda is assigned a role in the play.

- A)  $4/10$   
 B)  $3/10$   
 C)  $2/10$   
 D)  $5/10$   
 E) none of the others

2 MALES NOT ALEX  
 2 FEMALES NOT ZELDA  
 3 MALES NOT ALEX  
 1 Female NOT ZELDA

$$\frac{C(4,2)C(3,2) + C(4,3)C(3,1)}{C(5,2) \cdot C(4,2)} = \frac{6 \cdot 3 + 4 \cdot 3}{10 \cdot 6}$$

20. A box contains 5 balls with numbers 1 through 5. Two balls are drawn simultaneously and at random and the numbers on the balls are noted. Find the probability that the sum of the numbers is at least 4.

- A)  $22/25$   
 B)  $3/5$   
 C)  $9/10$   
 D)  $4/5$   
 E) none of the others

$$1 - \Pr(\text{SUM} \leq 3) = 1 - \Pr(\text{SUM} = 3)$$

$$= 1 - \Pr(1+2) = 9/10$$

$$\frac{2}{5 \cdot 4} = \frac{2}{20} = \frac{1}{10}$$

21. A menu in an Italian restaurant has 3 kinds of salads and 8 toppings for pizza. A student plans to order a meal consisting of a salad and a pizza with 3 different toppings. Find the number of different meals available.

- A)  $3 \times P(8,3)$   
 B)  $3^2 \times P(8,3)$   
 C) 59  
 D) 168  
 E) none of the others

$$3 \cdot C(8,3) = 3 \cdot \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} =$$

$$= 3 \cdot 56 = 168$$

22. A group of 6 students consists of 4 freshmen and 2 sophomores. Two students are selected at random. Find the probability that both are freshmen given that at least 1 is a freshman.

- A)  $3/5$   
 B)  $2/5$   
 C)  $3/7$   
 D)  $4/7$   
 E) none of the others

$$\Pr(BF | AIF) = \frac{\Pr(B \cap AIF)}{\Pr(AIF)} = \frac{\Pr(B)}{1 - \Pr(OF)}$$

$$= \frac{C(4,2)/C(6,2)}{1 - 1/C(6,2)} = \frac{6/C(6,2)}{\frac{C(6,2)-1}{C(6,2)}} = \frac{6}{6 \cdot 5/2 - 1} = \frac{6}{14}$$

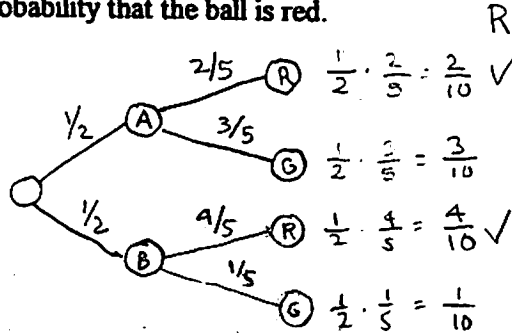
OR USE A TREE

23. Suppose A is the set of IU students who are seniors and B is the set of IU students who are Indiana residents. Which set describes those IU students who are seniors and are not Indiana residents?

- A)  $A \cup B'$   
 B)  $A \cap B'$   
 C)  $(A \cap B)'$   
 D)  $A' \cap B$   
 E) none of the others

24. Box A contains 2 red balls and 3 green balls and box B contains 4 red balls and 1 green ball. A box is selected at random, then a ball is drawn at random from that box and its color is noted. Find the probability that the ball is red.

- A)  $1/5$   
 B)  $3/5$   
 C)  $1/10$   
 D)  $2/5$   
 E) none of the others



$$\frac{2}{10} + \frac{4}{10} = \frac{6}{10} = \frac{3}{5}$$

25. A student has 6 tasks to do, and she plans to do 4 of them today and 2 tomorrow. In how many ways can she choose the 4 tasks to do today?

- A)  $P(6,4) \times P(2,2)$   
 B)  $P(6,4)$   
 C) 12  
 D) 15  
 E) none of the others

$$C(6,4) = C(6,2) = \frac{6!}{4!2!} = \frac{6 \cdot 5}{2} = 15$$