

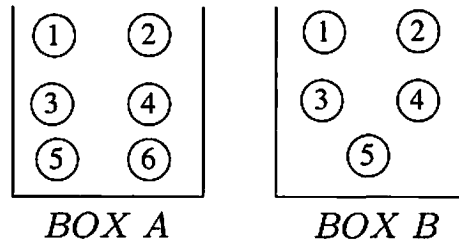
- 1) From a group of 6 employees (named Alex, Bob, Carol, Dorothy, Edward, and Fiona), one is to be selected for a \$2,000 bonus, one for a \$1,000 bonus, and one for a \$500 bonus. In how many ways can the bonuses be handed out? Note: No employee can receive two bonuses.

$$6 \cdot 5 \cdot 4 = 120$$

Example: One way is to give Alex the \$2,000 dollar bonus, Dorothy the \$1,000 bonus, and Edward the \$500 bonus.

- a) 60
 b) 120
 c) 144
 d) 128
 e) 96
 f) None of the above.

- 2) Box A contains 6 balls labelled 1 through 6. Box B contains 5 balls labelled 1 through 5. A ball is drawn at random from Box A, and then a ball is drawn at random from Box B. What is the probability that the sum of the two numbers drawn is 10?



Example: From Box A draw the ball labelled 6, then from Box B draw the ball labelled 3. The sum in this case is $6 + 3 = 9$.

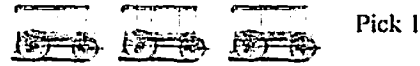
- a) $1/6$
 b) $7/36$
 c) $1/10$
 d) $1/15$
 e) $2/15$
 f) None of the above.

$6 \times 5 = 30$ possible outcomes

Only $(6,4), (5,5)$ yield a sum of 10

$$\frac{2}{30} = \frac{1}{15}$$

- 3) A wagon train master has to pick one of three wagons for a trip. Additionally he has to pick (for the trip) one of 4 available drivers, and 4 horses from a group of 5 horses. In how many ways* can he do this?



*Note: Order of choice does not matter - only which wagon, which driver, and which four horses matters.
 Example: One way is to pick wagon 2, driver 4, and horses 1,2,4, and 5.

- a) 14
 b) 100
 c) 88
 d) 12
 e) 60
 f) None of the above.

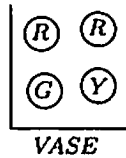
$$3 \cdot 4 \cdot C(5, 4) = 3 \cdot 5 \cdot 4 = 60$$

- 4) A box contains 3 red marbles and 2 green marbles. Two marbles are drawn at random from the box, one after another, without replacement. What is the probability that the second marble drawn is red?

- a) $1/2$
 b) $3/4$
 c) $1/4$
 d) $2/5$
 e) $3/5$
 f) None of the above.

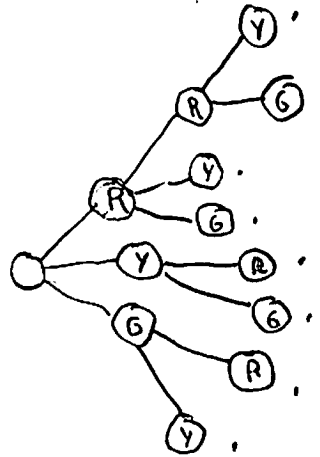
$$\frac{3}{5}$$

- 5) A vase contains two red marbles, one yellow, and one green marble. One marble after another is drawn out. The process is terminated as soon as marbles of two DIFFERENT colors have been drawn. An outcome consists of the record of the colors drawn in the order that they occurred. How many outcomes are possible? Hint: Draw a tree.



Example: RG represents the outcome where a red marble was selected, then a green.

- a) 6
 b) 4
 c) 8
 d) 9
 e) 7
 f) None of the above.



8 dots

- 6) 6 males and 5 females try out for the TV show "Dancing with the Stars." Two males and two females will be selected to appear on the show. In how many ways can this be done?

Example: One way is to select males 1 and 3, and females 1 and 4. Order of selection does not matter.

- a) 300
 b) 180
 c) 150
 d) 25
 e) 360
 f) None of the above.

$$C(6,2) C(5,2) = \frac{6 \cdot 5}{2} \cdot \frac{5 \cdot 4}{2} = 150$$

- 7) The Olympic Committee has to pick one of four cities (called A, B, C, and D) to host the 2020 Olympics. Cities A, B, and C are all equally likely to be chosen to host the event. City D is twice as likely to be chosen as city A. Find the probability that city A is chosen.

- a) $1/3$
 b) Cannot be determined
 c) $1/4$
 d) $2/3$
 e) $1/5$
 f) None of the above.

$$w_A + w_B + w_C + w_D = 1$$

$$w_A + w_A + w_A + 2w_A = 1$$

$$\Rightarrow w_A = 1/5$$

- 8) Three people are selected at random from a group of 5 males and 3 females. What is the probability of that all three are the same gender?

- a) $11/56$
 b) $11/336$
 c) $5/28$
 d) $10/336$
 e) $3/5$
 f) None of the above.

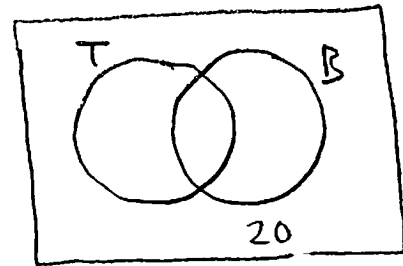
$$\frac{C(5,3) + C(3,3)}{C(8,3)} = \frac{11}{8 \cdot 7 \cdot 6 / 3!} = \frac{11}{56}$$

9) A group of 100 people are surveyed to see if they like the TV show "Two and a Half Men," and to see if they like the show "Big Bang Theory":

- 56 of them like the show "Two and a Half Men."
- 40 of them like the show "Big Bang Theory."
- 20 of them like neither show.

How many of the 100 people like both shows?

- a) 22
- b) 18
- c) 24
- d) 16**
- e) 20
- f) None of the above.



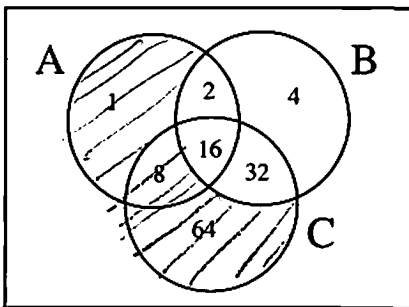
$$n(T \cup B) = 100 - 20 = 80$$

$$n(T \cup B) = n(T) + n(B) - n(T \cap B)$$

$$80 = 56 + 40 - n(T \cap B)$$

$$\Rightarrow n(T \cap B) = 16$$

10) Consider the following Venn diagram for sets A, B, C. Shown in this diagram are the number of elements in each indicated subset. How many elements are in the set $(A \cup C) \cap B'$?



- a) 89
- b) 123
- c) 73**
- d) 99
- e) 119
- f) None of the above.

$$1 + 8 + 64 = 73$$

- 11) Three fair (six sided) dice are rolled. What is the probability of getting 3 different numbers?

Example: The three numbers showing on the three dice are 5, 2, and 4. Five, 2, and 4 are three different numbers.

a) $2/3 = 24/36$

b) $15/216$

c) $1/4 = 9/36$

d) $5/9 = 20/36$

e) $111/216$

f) None of the above.

$$\frac{6 \cdot 5 \cdot 4}{6 \cdot 6 \cdot 6} = \frac{20}{36}$$

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- 12) How many 5 letter words can be formed using the letters AABCC that do NOT end with two C's? (Every word will have 2 A's, 2 C's, and 1 B.)

Examples: ACCBA, and CABAC do not end with 2 C's. The word BAACC does end with two C's.

a) 24

b) 27

c) 54

d) 57

e) 23

f) None of the above.

$$30 = \frac{5!}{2!2!} \text{ possible words}$$

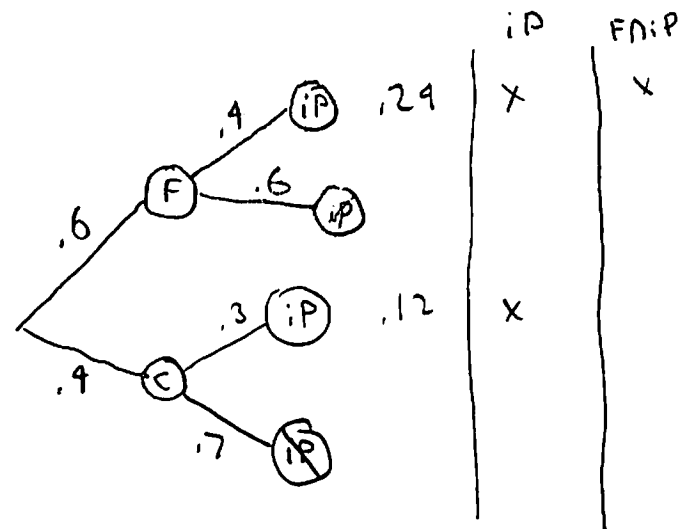
$$\underline{\quad\quad\quad} \text{CC} \quad 3 \text{ possible words}$$

2A's 1B

$$30 - 3 = 27$$

- 13) 60% of the cars at Sleazy Auto Sales are Fords, and 40% are Chevis. 40% of the Fords have iPod holders, and 30% of the Chevis have iPod holders. A car is selected at random from the lot, and it turns out to have an iPod holder. What is the probability that it is a Ford?

- a) .64
 b) .60
 c) $3/4 = 18/24$
 d) .72
 e) $2/3 = 24/36$
 f) None of the above.



$$Pr(F | iP) = \frac{Pr(F \cap iP)}{Pr(iP)} = \frac{.24}{.36} = \frac{2}{3}$$

- 14) Alex, Betty, Charlie, and Donna are finalists in a contest, and two of these four people are chosen at random to win a trip to the Galapagos Islands. Let X be the number of females that win a trip to the Galapagos Islands in this contest. Find $Pr(X = 1)$.

Alex and Charlie are men, and Betty and Donna are women.

- a) $3/4$
 b) $2/3$
 c) $1/4$
 d) $1/2$
 e) $5/8$
 f) None of the above.

$$Pr(X=1) = Pr(1M, 1W) \\ = \frac{2 \cdot 2}{C(4,2)} = \frac{4}{6} = \frac{2}{3}$$

- 15) Shown to the right are 5 boxes. If two of them are selected at random, what is the probability that the selected boxes have a side in common (i.e. are touching)?



a) $7/20$

b) $4/10$

c) $6/10$

d) $9/20$

e) $5/10$

f) None of the above.

$$1 \frac{4}{C(5,2)} = \frac{4}{10}$$

- 16) A small deck of four cards consists of two red cards and two black cards. Two cards are drawn from the deck at random and without replacement. What is the probability that both of the cards drawn are red given that at least one is red?

a) $2/5$

b) $3/8$

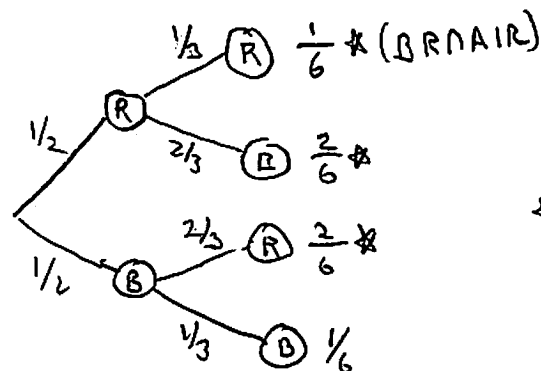
c) $1/3$

d) $1/4$

e) $1/5$

f) None of the above.

$$Pr(BR|AIR) = \frac{Pr(BR \cap AIR)}{Pr(AIR)} = \frac{1/6}{1/6 + 2/6 + 2/6} = 1/5$$



* = AIR

- 17) The table shown below gives the values of a random variable X and the density function of X . Unknown (until you figure it out) is the value of p . Find $E[X]$.

Value of X	Probability
-5	.20
-2	p
0	$2p$
5	.20
10	.15

$$3p = .45$$

$$p = .15$$

$$-5(\cancel{.2}) - 2(.15) + 5(\cancel{.2}) + 10(.15)$$

$$= 1.2$$

a) 1.20

b) 1.45

c) 1.30

d) 1.40

e) 2.30

f) None of the above.

- 18) A four letter word is spelled at random* using the letters ABBL. What is the probability that that word is BLAB?

From a list of all possible words that can be spelled using 2 B's, 1 A, and 1 L, one word is chosen at random. Each possible word only appears once on the list.

a) $1/6 = 2/12$

b) $1/12$

c) $1/8 = 3/24$

d) $1/24$

e) $1/4 = 3/12$

f) None of the above.

$$\frac{4!}{2!} \text{ words} = 12 \text{ words}$$

BLAB is one of them.

$$\frac{1}{12}$$

- 19) From a group of 6 employees (named Alex, Bob, Carol, Dorothy, Edward, and Fiona), one is to be selected for a \$2,000 bonus, one for a \$1,000 bonus, and one for a \$500 bonus. In how many ways can the bonuses be handed out if Carol receives one of the bonuses? Note: No employee can receive two bonuses.

Example: One way is to give Dorothy the \$2,000 dollar bonus, Carol the \$1,000 bonus, and Edward the \$500 bonus.

- a) 60
 b) 145
 c) 144
 d) 96
 e) 120
 f) None of the above.

Give Carol a prize. 3 ways.
 Select 1 of 5 remaining people
 for one of the other prizes 5 ways
 Select another person 4 ways
 $3 \cdot 5 \cdot 4 = 60$

- 20) A drawer contains 3 red socks and 2 blue socks. A sock is selected at random, its color is noted, and then it is replaced. This process is repeated 2 more times for a total of 3 draws. What is the probability that 1 red sock and two blue socks were drawn out (in any order)?

- a) $7/10$
 b) $1 - \frac{3 \cdot 2^2}{5^3}$
 c) $3/10$
 d) $\frac{3 \cdot 2^2}{5^3}$
 e) $3 \cdot \frac{3 \cdot 2^2}{5^3}$
 f) None of the above.

Bernoulli

$$3 \cdot \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}$$

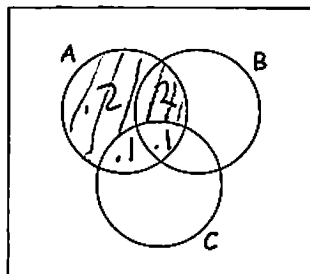
21) Suppose $A, B,$ and C are sets with:

$$Pr(A) = .6 \quad Pr(B) = .5 \quad Pr(C) = .5$$

$$Pr(A \cap B) = .3 \quad Pr(B \cap C) = .3$$

$$Pr(A \cap C) = .2 \quad Pr(A \cap B \cap C) = .1$$

Find $Pr(A \cap C')$.



a) .4

b) .3

c) .1

d) 0

e) .2

f) None of the above.

22) Suppose $Pr(A) = .4$ and $Pr(B) = .5$, where A and B are events in a sample space S . If A and B are independent, then what is $Pr(A \cup B)$?

a) .9

b) .6

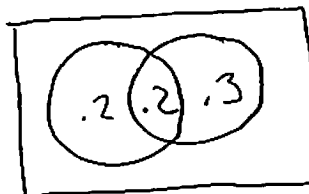
c) .7

d) 1

e) .8

f) None of the above.

$$Pr(A \cap B) = Pr(A) Pr(B) = .4 \cdot .5 = .2$$



23) Suppose A and B are two events with $Pr(A) = .3$ and $Pr(B) = .4$. If $Pr(A|B) = .5$, find $Pr(B|A)$.

a) .35

b) $1/2$

c) $1/3$

d) .4

e) $2/3$

f) None of the above.

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A \cap B)}{.4} = .5$$

$$\Rightarrow Pr(A \cap B) = .2$$

$$Pr(B|A) = \frac{Pr(A \cap B)}{Pr(A)} = \frac{.2}{.3}$$

24) A fair coin is flipped twice. You win:

- +\$6 if the result is two heads.
- +\$2 if the result is a heads and tail in any order.
- -\$4 if the result is two tails (i.e. lose \$4).

What are your expected winnings in dollars?

a) 1.25

b) 1.50

c) 1.35

d) 1.00

e) 1.20

f) None of the above.

$$6\left(\frac{1}{4}\right) + 2\left(\frac{1}{2}\right) + (-4)\left(\frac{1}{4}\right)$$

$$= 1.5 + 1 - 1 = 1.5$$

- 25) A vase contains 3 red flowers and 4 white flowers. Two flowers are selected at random, one after the other, without replacement. What is the probability that they are both white?

a) $8/21$

b) $12/21 = 4/7$

c) $6/21 = 2/7$

d) $5/42$

e) $7/21 = 1/3$

f) None of the above.

$$\frac{C(4,2)}{C(7,2)} = \frac{6}{7 \cdot 6 / 2} = \frac{2}{7}$$