1. In this problem, A, B, C are subsets of a universal set U. One of the following is always true. Which one?
   (A) $A \subset B \cup A'$
   (B) $A \cap B \subset B \cup C'$
   (C) $(A \cup B)' = A' \cup B'$
   (D) $(A \cap B \cap C') \cup (A' \cap B' \cap C) \subset A' \cap B' \cap C'$
   (E) $A' \cap B' \cap C' = \emptyset$
   (F) none of the above

$A \cap B \subset B \subset B \cup C'$

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2. For two events A and B we have $Pr[A'] = .71$, $Pr[B] = .43$, and $Pr[A \cup B] = .64$. Find $Pr[A \cap B]$.
   (A) .07
   (B) .43
   (C) .27
   (D) .08
   (E) .16
   (F) none of the above

$Pr(A') = .71 \Rightarrow Pr(A) = .29$

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$.64 = .29 + .43 - Pr(A \cap B)$

$.64 = .72 - Pr(A \cap B)$

$Pr(A \cap B) = .08$

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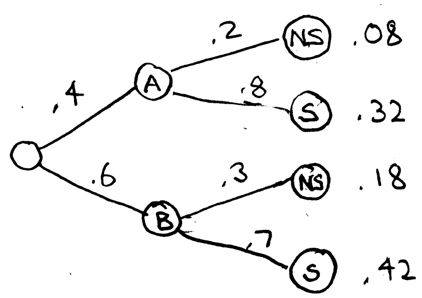
3. The *Lord* motor company produces 40% of its cars at plant A and the remainder at plant B. Of all cars produced at plant A, 20% do not have a spare tire, while 30% of the cars produced at B do not have a spare tire. A *Lord* car is purchased, and it does not have a spare tire. What is the probability that the car was produced at plant B?
   (A) 4/13
   (B) 16/37
   (C) 1/2
   (D) 21/37
   (E) 9/13
   (F) none of the above



$Pr(B|NS) = \frac{Pr(B \cap NS)}{Pr(NS)} = \frac{.18}{.18 + .08} = \frac{18}{26} =$

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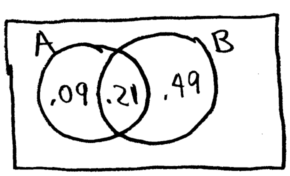
4. Two independent events A and B have probabilities $Pr[A] = .3$ and $Pr[B] = .7$. Find $Pr[A \cup B]$.
   (A) 1
   (B) .7
   (C) .79
   (D) .21
   (E) .49
   (F) none of the above



INDEPENDENCE $\Rightarrow$

$Pr(A \cap B) = Pr(A) \cdot Pr(B) = .3 \cdot .7 = .21$

NOW USE $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$= .3 + .7 - .21 = .79$

OR FILL IN VENN DIAGRAM

$.3 + .49 = .79$

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5. An unfair (six sided) die is such that the outcomes 1, 2, 3, 4 are equally likely, 5 is half as likely as 2, and 6 is four times as likely as 4. Find $Pr[1]$.
   (A) 2/17
   (B) 1/6
   (C) 1/17
   (D) 1/10
   (E) 17/4
   (F) none of the above

$w(1) = w(2) = w(3) = w(4)$ $\quad \frac{1}{2} w(2) = w(5)$

$w(6) = 4w(4)$

$w(1) + w(2) + w(3) + w(4) + w(5) + w(6) = 1$

$= 4w(1) + \frac{1}{2} w(1) + 4w(1) = 1$

$\Rightarrow \frac{17}{2} w(1) = 1 \Rightarrow w(1) = 2/17$

6. A Finite Math. class consists of 41 freshmen and 52 sophomores. The class decides that not enough homework is assigned, and a delegation of 5 is chosen *at random* to discuss the matter with the instructor of the class. Find the probability that no freshmen are included in the delegation.
   (A) $\frac{C(52,5)}{C(93,5)}$ ✓
   (B) 41!/93!
   (C) 0
   (D) $1 - \frac{C(52,5)}{C(93,5)}$
   (E) $\frac{C(41,5)}{C(93,5)}$
   (F) none of the above

ways to pick 5 with no F (ie. PICK 5 S)

$\frac{C(52,5)}{C(93,5)}$ ← # ways to pick any 5

7. A Bernoulli trial with success probability $p = .2$ is repeated *independently* 15 times. What is the probability that the second, fourth, and tenth of the tries were failures, but all the other tries were successes?
   (A) $C(15,12)(.2)^3(.8)^{12}$
   (B) $(.2)^{12}(.8)^3$ ✓
   (C) $(.2)^3$
   (D) $(.2)^3(.8)^{12}$
   (E) $1 - C(15,12)(.2)^3(.8)^{12}$
   (F) none of the above

SFSFSSSSSFSSSSS

The probability of this particular outcome is

$.2\cdot.8\cdot.2\cdot.8\cdot.2\cdot.2\cdot.2\cdot.2\cdot.2\cdot.8\cdot.2\cdot.2\cdot.2\cdot.2\cdot.2$

$= (.8)^3(.2)^{12}$

8. A multiple choice exam consists of 6 questions, and each question has 3 possible answers, exactly one of which is correct. Pif decides to pick the answer to each question at random. What is the probability that he will get precisely two correct answers?
   (A) 1/3
   (B) 80/243
   (C) 40/243
   (D) 16/729
   (E) 2/3
   (F) none of the above

Bernoulli

$C(6,2)\,(1/3)^2\,(2/3)^4$

$= 15\cdot\frac{2^4}{3^6} = 5\cdot\frac{2^4}{3^5} = 5\cdot\frac{16}{243} = \frac{80}{243}$

9. Mathieu has 4 white socks and 6 black socks in a drawer. He selects 2 socks at random (he's a little sleepy this morning) and puts them on. What is the probability that he wears socks of the same color?
   (A) 8/15
   (B) 24/45
   (C) 1/2
   (D) 1/3
   (E) 7/15
   (F) none of the above

ways to select 2W

$\frac{C(4,2) + C(6,2)}{C(10,2)}$ ← # ways to select 2B; ← # ways to select any 2 socks

$= \frac{6+15}{45} = \frac{21}{45} = 7/15$

OR: $Pr(\text{Same Color}) = 1 - Pr(1B+1W) = 1 - \frac{4\cdot 6}{C(10,2)} = 1 - \frac{24}{45}$

10. Cleopatra has 5 chains in a jewelry box: 4 silver chains worth \$40 each, and 1 gold chain worth \$2500. She selects a chain at random to wear for her midterm exam. What is the expected dollar value of the chain selected?
(A) \$1015
(B) \$532
(C) \$50.20
(D) \$432
(E) \$1030
(F) none of the above

Pr(Selecting silver chain) = 4/5
Pr (Gold chain) = 1/5

$\frac{4}{5} \cdot 40 + \frac{1}{5} \cdot 2500 = 32 + 500 = 532$

11. A class consists of 8 sophomores and 4 juniors. Two students are selected at random to form a committee. Find the expected number of sophomores on this committee.
(A) 2/3
(B) 1/3
(C) 1/2
(D) 4/3
(E) 1
(F) none of the above

Let X = # Sophomores.

$Pr(X=0) = Pr(2J) = \frac{C(4,2)}{C(12,2)} = \frac{6}{66}$

$Pr(X=1) = Pr(1S+1J) = \frac{8 \cdot 4}{C(12,2)} = \frac{32}{66}$

$Pr(X=2) = 1 - Pr(X=1) - Pr(X=0) = 1 - 38/66 = 28/66$

$6/66 \cdot 0 + 32/66 \cdot 1 + 28/66 \cdot 2 = 88/66 = 8/6 = 4/3$

12. A restaurant offers 4 kinds of salads, 6 main courses, and 1 dessert. A meal at this restaurant consists of a salad, a main course, and possibly a dessert. (Dessert is optional.) How many different meals can one order at this restaurant?
(A) $C(9,2)$
(B) $C(10,3)$
(C) 40
(D) 48
(E) 20
(F) none of the above

choices for main course

$4 \cdot 6 \cdot 2$ ← 2 choices dessert (yes or no)

choices for salad

$= 48$

13. A student has 3 volumes by K. Marx and 5 movies with the Marx brothers. His evening schedule consists of watching a movie, then studying one of the Marx volumes, and then again watching a movie, different from the first. How many different evening schedules are there?
(A) 12
(B) 60
(C) $3C(5,2)$
(D) 90
(E) 13
(F) none of the above

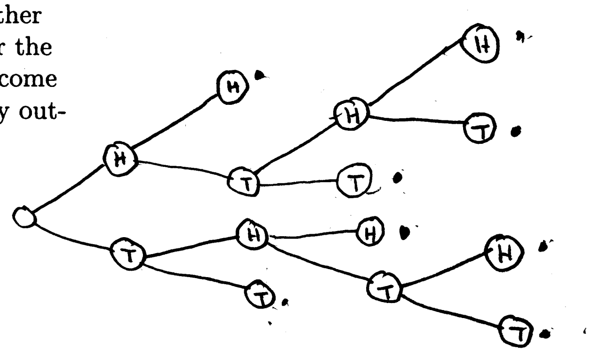
$5 \cdot 3 \cdot 4$ ← # choices 2nd movie

choices for book

choices for 1st movie

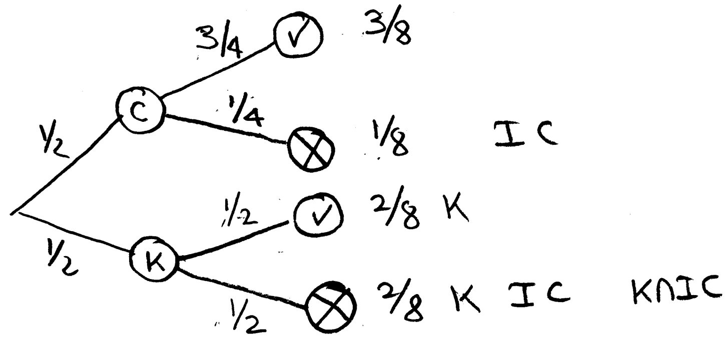
$= 60$

14. In an experiment a coin is tossed until either the same side comes up twice in a row, or the coin has been tossed four times. The outcome (H or T) of each toss is noted. How many outcomes are there for this experiment?
(A) 16
(B) 8
(C) 10
(D) 6
(E) this experiment never stops
(F) none of the above



8 dots

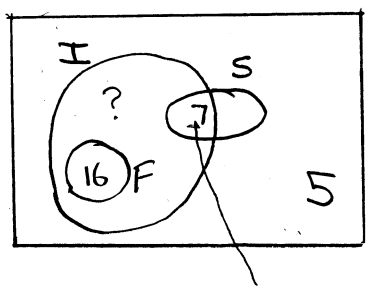
15. Bart has a *Cardin* calculator and a *Klein* calculator. The *Cardin* gives the correct answer 75% of the time, while the *Klein* gives the right answer 50% of the time. Bart needs to calculate 2×3, and he chooses a calculator randomly. Given that the answer he obtains is 5, what is the probability that he used *Klein*?
(A) 1/8
(B) 1/2
(C) 1/3
(D) 1/4
(E) 2/3
(F) none of the above



$$Pr(K|IC) = \frac{Pr(K\cap IC)}{Pr(IC)} = \frac{2/8}{2/8+1/8} = 2/3$$

16. In a class of 40 students there are 16 freshmen, 14 sophomores, 28 Indiana residents, and 5 students who are neither freshmen nor sophomores nor Indiana residents. All freshmen in this class are Indiana residents. How many Indiana residents are neither freshmen nor sophomores?
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6
(F) none of the above

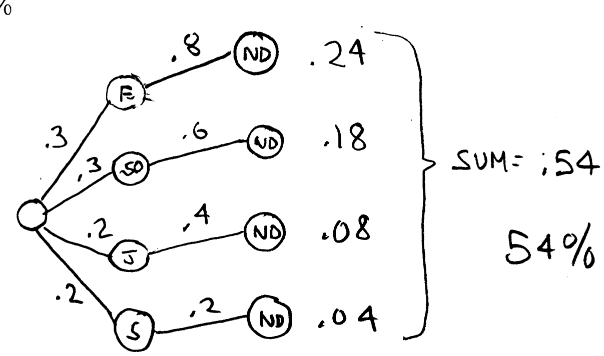
I = INDIANA RESIDENTS
NOTE: F ⊂ I



$n(I \cup S) = 40 - 5 = 35$
$= n(I) + n(S) - n(I\cap S)$
$= 28 + 14 - n(I\cap S)$
$\Rightarrow 35 = 42 - n(I\cap S)$
$\Rightarrow n(I\cap S) = 7 \Rightarrow 16+7+? = n(I) = 28$
$\Rightarrow ? = 5$

17. A survey at *IU* reveals that 20% of the freshmen, 40% of the sophomores, 60% of the juniors, and 80% of the seniors have made a decision about their future carreer. Given that 30% of all students are freshmen, 30% are sophomores, 20% are juniors, and 20% are seniors, what percentage of the whole student population have **not** made up their mind about their future carrer?
(A) 54%
(B) 46%
(C) 50%
(D) 67%
(E) not enough information given
(F) none of the above

PARTIAL TREE ⇒

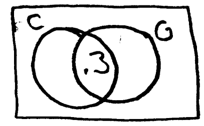


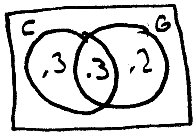
54%

18. The *Meré* space station has a computer which fails with probability .6, and an oxygen generator which fails with probability .5; the two kinds of failures occur independently. What is the probability that exactly one failure occurs?
(A) .5
(B) .6
(C) .7
(D) .32
(E) .68
(F) none of the above

Event of computer failure - C
Event of O_2 generator failure - G

$Pr(C \cap G) = Pr(C) \cdot Pr(G) = .6 \cdot .5 = .30$

$\Rightarrow$ 

$\Rightarrow$ 

$\Rightarrow$ .3+.2=.5

19. A universal set $U = \{1,2,3,4,5,6,7,8\}$ has subsets $A = \{1,3,4,6\}$, $B = \{1,2,6,7\}$ and $C = \{1,4,5,6\}$. Find the set $(A \cap C) \cup (C \cap B')$.
(A) $\{1,2,3,4,5,6,8\}$
(B) $\{1,2,4,5\}$
(C) $\{1,4,5\}$
(D) $\{1,4,5,6\}$
(E) $\{4\}$
(F) none of the above

$A \cap C = \{1,4\}$

$C \cap B' = \{4,5,6\}$

$(A \cap C) \cup (C \cap B') = \{1,4\} \cup \{4,5,6\} = \{1,4,5,6\}$

20. A panel of five directors votes on approval of a project. Approval is granted if at least three of the directors vote favorably. Each director votes either for or against; if the project is approved, the directors who voted in favor form the supervisory board for the project. How many different supervisory boards can be selected this way?
(A) 6
(B) 16
(C) 26
(D) 8
(E) 10
(F) none of the above

5 YES VOTES $\Rightarrow$ 1 WAY TO DO THIS (THEY ALL VOTE YES) $\Rightarrow$ 1 BOARD $\ast_1$

4 YES VOTES $\Rightarrow C(5,4) = 5$ WAYS $\Rightarrow$ 5 POSSIBLE BOARDS $\ast_2$

3 YES VOTES $\Rightarrow C(5,3) = \frac{5!}{3!2!} = 10$ WAYS

$\Rightarrow$ 10 POSSIBLE BOARDS $\ast_3$

$\ast_1 + \ast_2 + \ast_2 = 1+5+10 = 16$

21. The *Well-Done* hamburger place buys 100 hamburger patties infected with e-coli. E-coli in any patty survives cooking at *Well-Done* with a probability of .3. Considering the cooking of the 100 patties to be independent, what is the expected number of patties which will have no e-coli after cooking?
(A) 30
(B) 70
(C) 15
(D) 85
(E) 100
(F) none of the above

For each patty, the probability of being free of E. coli after cooking is .7

$E(X) = np$ for a Bernoulli process.

Let X = # patties with no E. coli after cooking. Then $p = .7$, $n = 100$

$np = 100 \cdot .7 = 70.$

22. You want to buy a three scoop cone at Baskin-Robbins (with the three scoops stacked one at the bottom, one in the middle, and one at the top, as usual). How many different choices do you have if you do care about the order of the flavors? (That is, Chocolate/Vanilla/Chocolate is different from Chocolate/Chocolate/Vanilla. Recall that there are 31 different flavors!)
(A) 93
(B) 3^{31}
(C) 31^3
(D) $P(31,3)$
(E) $C(31,3)$
(F) none of the above

31 choices for bottom scoop.
31 choices for middle scoop.
31 choices for top scoop.

$31 \cdot 31 \cdot 31 = (31)^3$

23. Four tennis players decide to play a doubles (two against two) match every day until all the possible team combinations are used. How many days will they play?
(A) 6
(B) 3
(C) 24
(D) 12
(E) 1
(F) none of the above

To select the teams for a day, choose 2 of the 4 players. Put the 2 on the same team & have them play the other 2.
$C(4,2)$ ways to choose 2.
$C(4,2)=6$

24. Jack and Jill have to fetch a pail of water which is now on top of a hill. On any given try, Jack has a probability of success of .5, while Jill's success probability is .9. They decide that Jack will try first and, in case he fails, Jill will also have one try. What is the probability that they will succeed; that is, what is the probability that at the end of this experiment, the pail will actually be fetched?
(A) .95
(B) .8
(C) .56
(D) .94
(E) it cannot be determined
(F) none of the above

$Pr(\text{Success}) = 1 - Pr(\text{Failure})$
$= 1 - Pr(\text{Jack fails} \,\&\, \text{Jill fails})$
$= 1 - Pr(\text{Jack F} \cap \text{Jill F})$ INDEPENDENT EVENTS
$= 1 - Pr(\text{Jack F}) \cdot Pr(\text{Jill F})$
$= 1 - (.5)(.1) = 1 - .05 = .95$

25. Professor Umbuggio has decided that there are too many yellow tulips in his garden. He wants to remove 5 of the 25 yellow tulips. What is the number of choices he has?
(A) $C(20,5)$
(B) $P(20,5)$
(C) $C(25,5)$
(D) 25^5
(E) 5^{25}
(F) none of the above

$C(25,5)$
from 25 objects select 5 without regard to order.