1) When making an online purchase of a computer, the buyer must configure the computer by selecting:

   1) The type of cpu.
   2) The size of the hard drive.
   3) The amount of RAM.
   4) Whether or not to install bluetooth.

   The buyer is free to select any of the 3 types of cpu, any of the 3 different sizes of hard drive, and any of the 4 RAM sizes offered. The bluetooth is either installed or not - the buyer selects "yes" or "no". How many possible configurations are there?

   $3 \times 3 \times 4 \times 2 = 72$

   a) 38

   b) 68

   c) 26

   d) 13

   e) 72

   f) None of the above.

---

2) Three rooms have to be painted. Each room is to be painted one color: red, green, or blue. In how many ways can this be done so that more than one color is used?

   Example1: Paint room 1 green, room 2 blue, and room 3 green.
   Example 2: Paint room 1 blue, room 2 green, and room 3 green.
   NOT A WAY: Paint all three rooms red.

   USING ANY COLORS.

   $3 \times 3 \times 3 = 27$

   BUT RRR, GGG, BBB ARE NOT ALLOWED

   ANSWER: $27 - 3 = 24$

   a) 26

   b) 21

   c) 9

   d) 12

   e) 24

   f) None of the above.

3) The next flight out needs a pilot, a copilot, and a flight engineer. There are 9 personnel (all equally qualified) available to fill these positions. In how many ways can these positions be filled?

Note: Who gets which position matters.

a) 84

b) 723

c) 645

d) 504

e) 729

f) None of the above.

9 CHOICES

8 CHOICES

7 CHOICES

P CP FE

$9 \cdot 8 \cdot 7 = \underline{\underline{504}}$

---

4) Three poker chips are worth \$2 and two poker chips are worth \$1. A process consists of selecting these chips one at at time without replacement until either there are no chips left or the total value of the chips that have been selected is \$4 or more. An outcome of this process is a record of the value of the chip selected on each draw*. How many records (or outcomes) are possible?

*Example: One record (or outcome) is 122 indicating that a \$1 chip was selected, then a \$2 chip was selected, then a \$2 chip. The process terminated since $1 + 2 + 2 \geq 4$.

PLACE A DOT • ON EACH BRANCH THAT TERMINATES

a) 11

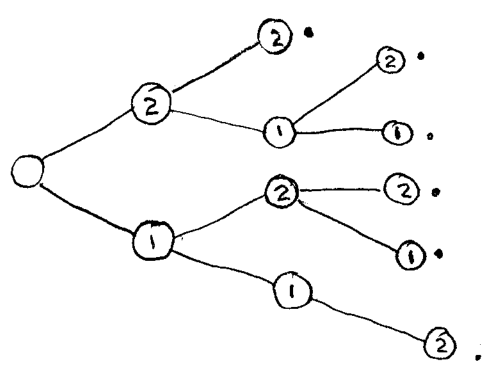
b) 6

c) 8

d) 4

e) 7

f) None of the above.



6 DOTS

ANSWER: 6

5) A drawer contains 5 red socks and 4 blue socks. Three (3) socks are taken from the drawer, at random, without replacement. What is the probability of getting 2 red socks and 1 blue sock?

a) 5/42

b) 1/6

c) 2/15

d) 10/21

e) 23/84

f) None of the above.

WAYS TO PICK 2R

WAYS TO PICK 1 B

$$\frac{C(5,2)\,C(4,1)}{C(9,3)} = \frac{10 \cdot 4}{9 \cdot 8 \cdot 7/3 \cdot 2 \cdot 1} = \frac{10}{21}$$

WAYS TO PICK 3 OF 9

ORDER DOESN'T MATTER, SO USE COMBINATIONS.

---

6) Two hundred (200) mathematicians attend a math conference. Forty five (45) of these mathematicians have buck teeth (**BT**). Eighty five (85) have skinny legs (**SL**). One hundred and ten (110) of these mathematicians don't have buck teeth and don't have skinny legs. How many have buck teeth and skinny legs?

a) 110

b) 50

c) 55

d) 40

e) 90

f) None of the above.

$$(BT \cup SL)^c = \overbrace{(BT)^c \cap (SL)^c}^{\text{DON'T HAVE BT AND DON'T HAVE SL}}$$

$$n(BT \cup SL) = 200 - n((BT \cup SL)^c)$$
$$= 200 - 110 = 90$$

$$n(BT \cup SL) = n(BT) + n(SL) - n(BT \cap SL)$$
$$90 = 45 + 85 - n(BT \cap SL)$$
$$\Rightarrow n(BT \cap SL) = \underline{\underline{40}}$$

7) Shown to the right are 6 boxes (i.e. 6 squares), 4 in the top row and 2 in the bottom row. If two of these six boxes are selected at random, then the probabliity that they are both in the same row is $\dfrac{X}{C(6,2)}$.

Find X:

a) 13

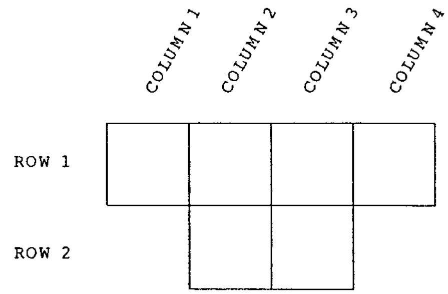
b) 12

c) 8

d) 10

e) 7

f) None of the above.



WAYS TO CHOOSE 2 SQUARES IN ROW1 = C(4,2) (choose 2 of the 4)

WAYS TO CHOOSE 2 SQUARES IN ROW2 = 1 (choose both).

WAYS TO CHOOSE 2 SQUARES IN SAME ROW:

$C(4,2) + 1 = 6 + 1 = \underline{\underline{7}}$ ← ANSWER

$\dfrac{7}{C(6,2)}$ = Pr (2 IN SAME ROW)

C(6,2) ← # WAYS TO CHOOSE ANY 2 OF THE 6 □'s.

---

8) Suppose A and B are <u>independent</u> events with $Pr[A \cap B] = .2$ and $Pr[B] = .4$. Find $Pr[A \cap B']$.

a) .45

b) .4

c) .2

d) .5

e) .3

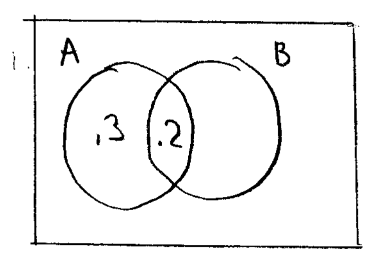
f) None of the above.

$Pr(A \cap B) = Pr(A) \cdot Pr(B)$ (independence)

$.2 = Pr(A) \cdot .4$

$\Rightarrow Pr(A) = .5$

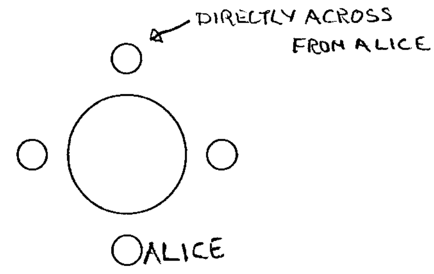
$\Rightarrow$



$.3 + .2 = .5$

$\Rightarrow Pr(A \cap B') = \underline{\underline{.3}}$

9) Shown to the right is a circular table with 4 chairs (shown as four small circles) arranged around it. Alice, Betty, Cindy, and Danica are seated at the table at random (one person to a chair). What is the probability that Alice and Betty are sitting directly across the table from one another?



a) 1/4

b) 1/2

c) 1/6

d) 1/3

e) 2/3

f) None of the above.

SEAT ALICE FIRST. ASSUME SHE ENDS UP RIGHT HERE (IF NOT, ROTATE PICTURE AND MAKE IT SO).

NEXT SEAT BETTY. ONE OF THE THREE AVAILABLE SEATS IS DIRECTLY ACROSS FROM ALICE.

ANSWER: 1/3.

---

10) You have 10 employees. How many ways can you select 2 of them for a trip to New York and one of them for a trip to Chicago? No employee gets to go to both cities.

Example: Send employees 5 and 7 to NY, and send employee 4 to Chicago. Note that this is the same as sending 7 and 5 to NY and 4 to Chicago.

a) 360

b) 224

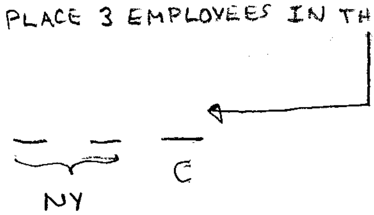
c) 53

d) 540

e) 98

f) None of the above.

PLACE 3 EMPLOYEES IN THESE 3 SLOTS



C(10,2) WAYS TO PICK 2 FOR NY, THERE ARE NOW 8 EMPLOYEES LEFT TO PICK FROM FOR CHICAGO, PICK ONE. THERE ARE 8 WAYS TO DO THIS.

ANSWER:

$C(10,2)\cdot 8 = \frac{10\cdot 9}{2}\cdot 8 = 360$

11) How many 7 letter "words" can be formed using the letters CCCCXXX?

Example: One such word would be CXXCCCX. Another word is XCCXCXC.

a) 210

b) 5040

c) 70

d) 35

e) 140

f) None of the above.

METHOD 1:

$C_1 C_2 C_3 C_4 X_1 X_2 X_3 \longrightarrow 7!$ possible words

$$\frac{7!}{4!\,3!} = \frac{7 \cdot 6 \cdot 5}{3!} = 35$$

$3!$ ← #WAYS TO ORDER $X_1 X_2 X_3$

$4!$ ← #WAYS TO ORDER $C_1 C_2 C_3 C_4$

METHOD 2:

7 SLOTS

_ _ _ _ _ _ _

CHOOSE 4 OF THE SLOTS FOR C's.

C(7,4) WAYS TO DO THIS: C(7,4) = 35.

FILL THE REST OF THE SLOTS WITH X'S.

1 WAY TO DO THIS: ANSWER: $35 \times 1 = \underline{\underline{35}}$

---

12) The table shown below gives the values of a random variable X and the density function for X. Unknown is the value of p_1. Find the value of $E(X)$.

Value of X	Probability
−50	.10
−10	p_1
0	.20
10	.25
50	.20

a) 10

b) 6.5

c) 5

d) -10

e) 12.5

f) None of the above.

$.1 + p_1 + .2 + .25 + .20 = 1 \Rightarrow p_1 = .25.$

$E(X) = .1(-50) + .25(-10) + 0 + .25(10) + .2(50)$

$= .1(50) = \underline{\underline{5}}$

13) A hat contains 4 white slips of paper and 2 red slips of paper. A slip is drawn at random from the hat, and its color is noted. Then it is REPLACED. This process is repeated two more times - for a total of 3 draws with replacement after each draw. What is the probability that two white slips and one red slip were drawn (in any order)?

a) 23/27

b) 4/9

c) 4/27

d) 3/5

e) 12/27

f) None of the above.

BOTH CORRECT

THIS IS A BERNOULLI PROCESS.

$Pr(W) = 2/3 \quad Pr(R) = 1/3$.

$Pr(2W + 1R) = C(3,2)\,(2/3)^2\,(1/3)^1$

$= 3 \cdot \frac{4}{3 \cdot 3 \cdot 3}$

$= \underline{\underline{4/9}}$

---

14) An vase contains 3 red flowers and 4 white flowers. Two flowers are selected at random, one after the other, without replacement. If the selected flowers are both the same color, what is the probability that they are both red?

a) 2/7

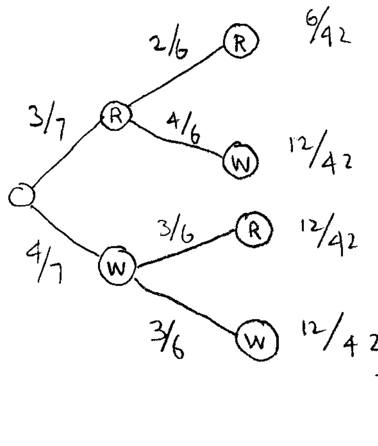
b) 1/3

c) 40/49

d) 4/7

e) 9/49

f) None of the above.



$Pr(BR \mid BSC) = \frac{Pr(BR \cap BSC)}{Pr(BSC)} =$

BR	BSC	BR∩BSC
X	X	X
	X	
6/42	$\frac{6}{42} + \frac{12}{42}$	

$= \frac{6/42}{6/42 + 12/42}$

$= \frac{6}{18} = \underline{\underline{1/3}}$

15) On any given day, Joe arrives late to work with probability 1/3. If he arrives late, he will leave work early with probability 3/4. If he does not arrive late, he will leave early with probability 1/2. Given that he leaves work early, what is the probablity that he arrived late to work that day?

$$Pr(AL|LE) = \frac{Pr(AL \cap LE)}{Pr(LE)} = \frac{3/12}{3/12 + 4/12} = \underline{\underline{3/7}}$$

a) 1/2

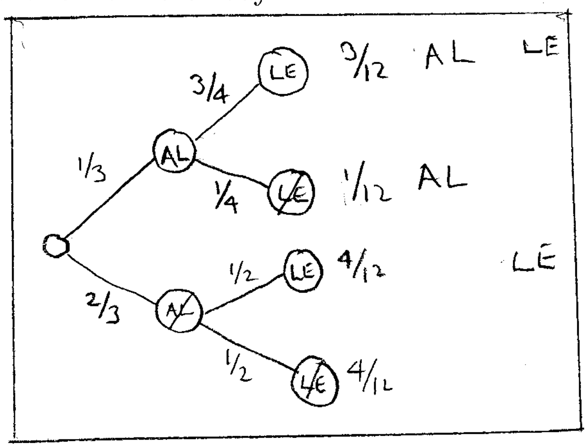
b) 1/3

c) 7/12

d) 3/7

e) 1/4

f) None of the above.



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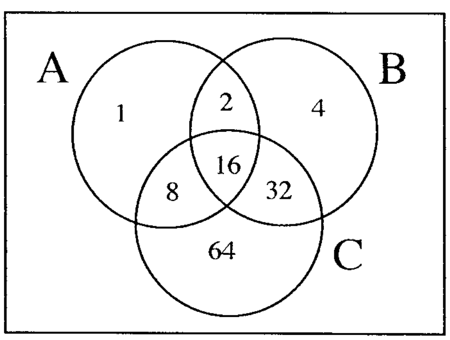
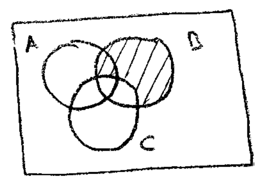
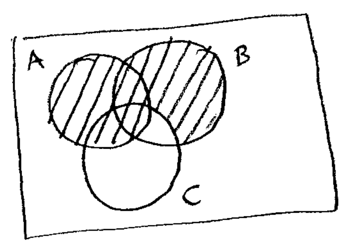
16) Consider the following Venn diagram for sets A, B, C. Shown in this diagram are the number of elements in each indicated subset. How many elements are in the set $(C' \cap B) \cup A$?



$C' \cap B \sim$



NOW TAKE THE UNION WITH A:



$\underbrace{1+2+16+8}_{A} + 4 = \underline{\underline{31}}$

a) 33

b) 38

c) 31

d) 1

e) 123

f) None of the above.

17) An unfair coin is flipped 20 times. On each toss, the probability of getting a heads is .7. Let X = # of heads - # tails* that occur in the 20 tosses. Find $E(X)$.

*Restated: X is the number of heads minus the number of tails.

a) 8

b) 14

c) .4

d) 0

e) 17

f) None of the above.

LET X_H = # HEADS

X_T = # TAILS.

$E(X_H - X_T) = E(X_H) - E(X_T)$

$= 20 \cdot .7 - 20 \cdot .3$

BERNOULLI RANDOM VARIABLE
20 TRIALS PR(SUCCESS) = .7

$= 20(.4) = \underline{\underline{8}}$

---

18) A bowl contains 6 red marbles and 3 blue marbles. Three marbles are drawn from the bowl, at random, one after another, without replacement. What is the probability of getting a red marble on the first draw, then a blue on the second, then a red on the third draw?

a) 36/56 = 9/14

b) 4/27

c) 10/56 = 5/28

d) 15/56

e) 12/56 = 3/14

f) None of the above.

WAYS TO DRAW 3 OF THE 9 KEEPING TRACK OF ORDER:

$9 \cdot 8 \cdot 7$.

WAYS TO DRAW R, THEN B, THEN R

$6 \cdot 3 \cdot 5$

ANSWER:

$$\frac{6 \cdot 3 \cdot 5}{9 \cdot 8 \cdot 7} = \underline{\underline{\frac{5}{28}}}$$

ALTERNATE WAY: "USE A TREE"

$$\frac{6}{9} \cdot \frac{3}{8} \cdot \frac{5}{7}$$

Pr(R on 1st draw)

Pr(B on 2nd draw)

Pr(R on 3rd draw)

19) A box contains red, green, and blue blocks. For every red block in the box there are two green blocks in the box (i.e. there are twice as many green blocks as red). For every blue block there are three green blocks. A block is selected at random from the box. What is the probability that it is NOT red?

a) 7/10

b) 5/7

c) 2/3

d) 8/11

e) 3/4

f) None of the above.

SUPPOSE THERE ARE 3 RED BLOCKS
THEN THERE 6 GREEN BLOCKS
+ THERE ARE 2 BLUE BLOCKS

3+6+2 = 11 BLOCKS

8 ARE NOT RED.

ANSWER: 8/11.

OR:

LET R = # red blocks, G = # green, B = # blue

$G = 2R$, $G = 3B \Rightarrow B = G/3 = \frac{2R}{3}$

TOTAL # BLOCKS:

$$R + G + B = R + 2R + 2R/3 = 11/3\, R$$

TOTAL # BLOCKS THAT ARE NOT RED:

$$G + B = 2R + \frac{2R}{3} = 8/3\, R$$

ANSWER: $\frac{8/3\, R}{11/3\, R} = 8/11$ $\left(\frac{\#\text{ NOT REDS}}{\#\text{ BLOCKS TOTAL}}\right)$

---

20) A group of five kids line up at random to get into a movie theatre. Two of the five kids are 8 years old and three are 9 years old. What is the probability that the two 8 year olds are first in line (i.e the 9 year olds are behind them)?

a) 5/20

b) 10/20

c) 6/20

d) 8/20

e) 2/20

f) None of the above.

1. # WAYS TO LINE UP: 5!
(ORDER MATTERS)

2. # WAYS TO PUT 2 EIGHT YEAR OLDS IN POSITIONS 1 AND 2: 2!

3. # WAYS TO PUT 3 NINE YEAR OLDS IN POSITIONS 3,4,+5: 3!

4. # WAYS TO LINE THE 5 KIDS UP WITH 8 YEAR OLDS FIRST IN LINE:

$$2! \cdot 3! = 12$$

ANSWER: $\frac{2! \cdot 3!}{5!} = \frac{2}{5 \cdot 4} = \frac{2}{20}$

21) There are 6 red socks and 3 black socks in a drawer. Three of the socks are drawn out at random, one after another, without replacement. What is the probability that at least one sock of each color was drawn?

For your convenience, all fractions are listed in reduced form as well as a form that may or may not appear in your calculations.

a) 483/504 = 23/24

b) 64/84 = 16/21

c) 503/504

d) 63/84 = 3/4

e) 384/504 = 16/21

f) None of the above.

$$\frac{\overbrace{C(6,2)C(3,1)}^{\text{\# WAYS TO DRAW 2R+1B}} + \overbrace{C(6,1)C(3,2)}^{\text{\# WAYS TO DRAW 1R+2B}}}{\underbrace{C(9,3)}_{\text{\# WAYS TO SELECT 3 OF 9 SOCKS W/O ORDER}}}$$

$$= \frac{6\cdot 5/2!\cdot 3 + 6\cdot 3}{9\cdot 8\cdot 7/3!}$$

$$= \frac{45+18}{84} = \frac{63}{84} = \underline{\underline{\frac{3}{4}}}$$

---

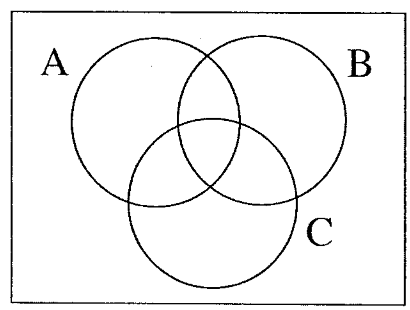
22) Suppose A,B, and C are sets with:

$n(A) = 100$ $\quad n(B) = 80$ $\quad n(C) = 70$

$n(A \cap B) = 20$ $\quad n(B \cap C) = 15$

$n(A \cap B \cap C) = 5$ $\quad n(A \cup B \cup C) = 190$

Find $n(A \cup B)$.



a) 130

b) 150

c) 160

d) 120

e) 135

f) None of the above.

$$n(A\cup B) = n(A) + n(B) - n(A\cap B)$$
$$= 100 + 80 - 20$$
$$= \underline{\underline{160}}$$

23) Three 200 pound people and one 100 pound person wait for an elevator. When it arrives, two of these four people are selected at random to get on the elevator. What is the **expected total weight** of these **two** passengers?

a) 275

b) 350

c) 325

d) 312.5

e) 400

f) None of the above.

METHOD I

X = TOTAL WEIGHT

X	Pr
300	
400	1/2

$Pr(X=400) = Pr(2 \times (200))$

$= \frac{C(3,2)}{C(4,2)} = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow$

X	Pr
300	1/2
400	1/2

$\Rightarrow 300 \cdot \frac{1}{2} + 400 \cdot \frac{1}{2} = \underline{\underline{350}}$

---

24) A password consists of 5 characters in a row. Three of these characters must be a 1, 2, and/or 3. Two of these characters must be an a, b, c, and/or d. How many passwords are possible?

Examples: ac332, 3c2d3, 123ab, 1bb11.

a) $10 \cdot 3^3 \cdot 4^2 = 4320$

b) $5! = 120$

c) $10 \cdot 5! = 1200$

d) $20 \cdot 3! \cdot 4! = 2880$

e) $10 \cdot 3! \cdot 4! = 1440$

f) None of the above.

_ _ _ _ _

① SELECT 3 SLOTS FOR NUMBERS

② C(5,3) WAYS TO DO THIS.

③ THOSE 3 SELECTED SLOTS CAN BE FILLED 3·3·3 WAYS

④ FILL THE REMAINING TWO SLOTS WITH LETTERS: 4×4 WAYS.

ANSWER:

$C(5,3) \cdot 3^3 \cdot 4^2 = \underline{10 \cdot 3^3 \cdot 4^2}$

25) Using 3 of the 7 letters ABCDEFR, how many words* can be formed that have the letter R in them? Note: A letter may NOT be used twice.

*Here are 4 such words: DFR, RDF, ARC, CRA.

a) 33

b) 210

c) 90

d) 120

e) 30

f) None of the above.

$7 \cdot 6 \cdot 5$ WORDS POSSIBLE WITH NO RESTRICTION

$6 \cdot 5 \cdot 4$ WORDS WITH NO R.

ANSWER: $7 \cdot 6 \cdot 5 - 6 \cdot 5 \cdot 4$

$= (7-4)(6 \cdot 5) = \underline{90}.$